Project No 518294 SES6

CASES

Cost Assessment of Sustainable Energy Systems

Instrument: Co-ordination Action

Thematic Priority: Sustainable Energy Systems

DELIVERABLE D.12.1
[WP12 Report on uncertainties and their effects]

Due date of deliverable: 30 September 2007
Actual submission date: 18 September 2007

Start date of project: 1st April 2007 Duration: 30 months

Organisation name of lead contractor for this deliverable: ECN

Revision: FEEM

Project co-funded by the European Commission within the Sixth Framework Programme (2002-2006)		
Dissemination level		
PU	Public	X
PP	Restricted to other programme participants (including the Commission Services)	
RE	Restricted to a group specified by the consortium (including the Commission Services)	
CO	Confidential, only for members of the consortium (including the Commission Services)	

Assessment of Uncertainties

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Abstract

A simple and transparent method for the uncertainty analysis of damage cost estimates is presented and applied to the damage cost of the classical air pollutants. The uncertainty is characterized in terms of geometric standard deviations σ_g and multiplicative confidence intervals: if a cost has been estimated to be μ_g (geometric mean ≈ median) with geometric standard deviation, the probability is approximately 68% that the true value is in the interval $[\mu_g/\sigma_g, \mu_g.\sigma_g]$ and 95% that it is in $[\mu_g/\sigma_g^2, \mu_g.\sigma_g^2]$. For NO_x, SO_2 and PM_{10} σ_g is about 3. For CO_2 and climate change we estimate the uncertainty on the basis of a literature review. The literature review indicates a σ_g of about 5.

The effect of the uncertainties on several policy choices is evaluated, in particular on the optimal levels for national emission ceilings. We also evaluate the benefit of reducing the uncertainties by further research (to help identify the priorities for such research).

1. Introduction

The objectives of this report are:

- To evaluate the uncertainties of the costs (both private and external) estimated in this project;
- To evaluate the effect of these uncertainties on policy decisions, e.g. choice of energy technologies, choice of emission limits, and the resulting emission levels;
- To evaluate the social costs if the wrong policy choices are made because of errors or uncertainties in the estimation of the costs estimated in this project;
- To evaluate the benefit of reducing the uncertainties by further research (to help identify the priorities for such research).

The uncertainties of environmental damages are far too large for the usual error analysis of physics and engineering (using only the first term in a Taylor expansion). Rigorous systematic assessment of the uncertainties is difficult and few studies have attempted it. Most merely indicate an upper and a lower value, but based on the range of just one input parameter or by simply combining the upper and lower bounds of several inputs, without taking into account the combination of uncertainties (e.g. of atmospheric dispersion, dose-response function and monetary valuation). Many damage assessments involve so many different inputs that an analytical solution was usually not considered, and of the uncertainty analyses that have been done, almost all use Monte Carlo techniques and numerical calculations [see e.g. Morgan et al 1984, and Morgan & Henrion 1990]. The Monte Carlo approach is powerful, capable of treating any problem, but it is computationally intensive and the result is "black box": it is difficult to see how important each of the component uncertainties is or how the result would change if a component uncertainty changes – especially for a reader who does not have access to the details of the calculations.

As a simple and transparent alternative Rabl and Spadaro [1999] have developed an analytic approach based on lognormal distributions. The justification lies in the observation that the calculation involves essentially a product of factors, and that the resulting uncertainty of the product is approximately lognormal for most damage costs of pollution. Thus it suffices to specify geometric mean and geometric standard deviations, or equivalently, multiplicative confidence intervals about the geometric mean (which is usually close to the median for damage costs). They can be interpreted in terms of multiplicative confidence intervals of the lognormal distribution: if a cost has been estimated to be μ_g (geometric mean ≈ median) with geometric standard deviation σ_g, the probability is approximately 68% that the true value is in the interval $[\mu_g/\sigma_g, \mu_g.\sigma_g]$ and 95% that it is in $[\mu_g/\sigma_g^2, \mu_g.\sigma_g^2]$. Following the practice of the physical sciences we show error bars corresponding to 1 standard deviation, unlike epidemiology and the social sciences where 95% confidence intervals (≈ 2 standard deviations) are usually reported.

Compared to a Monte Carlo analysis, this analytic approach yields typical answers that are easy to apply and communicate; the calculation is simple enough to allow the reader to modify the assumptions and see the consequences. Furthermore, the analytic approach can be coupled with Monte Carlo results for certain steps of the impact pathway analysis, thus combining the best features of each method; that has been demonstrated by Spadaro and Rabl [2007] for the dispersion of the classical air pollutants.

Whatever the method, an assessment of the uncertainties of damage costs must begin with a detailed examination of the uncertainties of each of the inputs to the impact pathway analysis, to estimate standard deviation and shape of the probability distribution of their uncertainties. This involves expert judgment with its unavoidably subjective aspects. These component uncertainties are then combined to obtain the total uncertainty of the damage cost. In Section 2 we update the original estimates of Rabl & Spadaro [1999] and show the results for mortality, the most important impact of the classical air pollutants (NO_x, PM, and SO_2). Of course the total damage cost of these pollutants is the sum over all impact, for instance health and agricultural losses. Spadaro and Rabl [2007] have shown that for typical cases the distribution of a sum of lognormally distributed damage costs is also approximately lognormal; they have also developed a simple approximate method for estimating the geometric mean and geometric standard deviation of the distribution of the sum.

This method can readily be used for most pollutants, not only NO_x, PM, and SO_2 but also toxic metals and dioxins. For greenhouse gases, by contrast, we have not been able to carry out an analogous analysis because we find the problem too complex, with too many different impact types and too many possible damage mechanisms. A thorough analysis, with detailed examination of the uncertainties of each of the inputs, would require mastery of models such as FUND or PAGE, clearly beyond the scope of the present project. For that reason we use a simpler global approach for greenhouse gases, and the presentation is different from the classical air pollutants.

The effects of the uncertainties on policy choices are explored in Section 4: in Section 4.1 the effect on different levels of internalization, in Section 4.1 the effect on emission ceilings. Finally, in Section 5, we evaluate the benefit of reducing the uncertainties by further research.

2. Uncertainties for PM, NOx and SO2

2.1. Methodology

The damage cost C for a single impact or health end point due to a pollutant is essentially a product of uncorrelated variables, as explained in the paper of Spadaro and Rabl [2007]. That is obviously the case if one does an approximate calculation using the "uniform world model" (UWM)

$C = p \rho s_{CR} / v_{dep}$	(1)

where p = cost per case ("price") [€/case],
s_{CR} = CRF slope [(cases/yr)/(pers·(µg/m^3))],
ρ = average population density [pers/km^2] within 1000 km of source, and
v_{dep} = deposition velocity of pollutant (dry + wet) [m/s].
The UWM is exact (because of the conservation of matter) in the limit where the distribution of either the sources or the receptors is uniform and the key atmospheric parameters are the same everywhere. In practice the agreement with detailed models is usually within a factor of two for primary pollutants and stack heights above 50 m. For secondary pollutants such as nitrates and sulfates the variation with stack height is negligible, and the agreement between UWM and detailed models is better than a factor of two. It yields typical, rather than site-specific results and is therefore relevant for policy applications where the sites are not known in advance.

But even for a detailed site-specific calculation using computer programs such as EcoSense, the key elements of the calculation enter in multiplicative fashion. Looking at a product z of uncorrelated variables x_i

$$z = x_1\, x_2\, x_3 \ldots x_n \tag{2}$$

and taking the logarithm, one sees that the familiar rules for the mean and standard deviation of a sum can be applied to the logarithms. The mean of the logarithm of a random variable is the logarithm of the geometric mean μ_g; specifically, if p(z) is the probability distribution of z, the geometric mean is given by

$$\ln(\mu_{gz}) = \int_0^\infty p(z)\ln(z)\,dz \tag{3}$$

Since the mean of ln(μ_{gz}) is the sum of the logarithms of the geometric means μ_{gxi} of the x_i, μ_{gz} is given by the product

$$\mu_{gz} = \mu_{gx1}\,\mu_{gx2} \ldots\ \mu_{gxn} \quad . \tag{4}$$

Let us now define the geometric standard deviation σ_{gz} as

$$[\ln(\sigma_{gz})]^2 = \int_0^\infty p(z)\,[\ln(z) - \ln(\mu_{gz})]^2\,dz \tag{5}$$

and analogously for the x_i. Assuming independence of the distributions one finds that the geometric standard deviation σ_{gz} of the product z is given by

$$[\ln(\sigma_{gz})]^2 = [\ln(\sigma_{gx1})]^2 + [\ln(\sigma_{gx2})]^2 + \ldots + [\ln(\sigma_{gxn})]^2 \quad . \tag{6}$$

This is the key formula for our estimation of the uncertainties. One needs to estimate the geometric standard deviations σ_{gxi} for each of the factors x_i in the calculation and combine them according to this equation. The relative importance of each uncertainty source is immediately obvious: because of the quadratic combination of terms, terms with small σ_{gxi} make a negligible contribution to the total uncertainty.

2.2. Total σ_g of Damage Costs

Based on a review of the literature and on expert judgment, we have estimated the geometric standard deviations σ_{gxi} for each of the key parameters that enter the calculation of mortality, which is by far the dominant impact; the results for other health endpoints are essentially the same. Since the other impacts of the classical air pollutants, namely crop losses and damage to buildings, are very small compared to the health impacts, their uncertainty makes a negligible contribution (again because of the usual quadratic combination of terms).

Table 1 shows our assumptions for the component uncertainties and the result for the damage cost for mortality. Of course such choices involve expert judgment with their inevitable subjective aspects. However, it is easy for the reader to make different choices and calculate the corresponding σ_g. For sulfates and nitrates ExternE assumes the same CRFs as for PM (apart from an overall scale factor), therefore the contributions to the uncertainty are the same for each of these pollutants, with fairly insignificant differences due to

i) atmospheric dispersion and chemistry (we assume different geometric standard deviations for PM, NO_x and SO_2).
ii) the toxicities of primary PM, sulfates and nitrates relative to ambient PM_{10}, as discussed at the end of Section 3.2 of Spadaro and Rabl [2007].

The resulting geometric standard deviations are 2.78 for primary PM, 3.26 for SO_2 and 3.39 for NO_x. The distribution of a product is exactly lognormal if each of the factors is lognormal. In practice it is sufficient for the factors with the largest widths to be approximately lognormal, a condition satisfied in the present case. Thus lognormality for the distribution of the result is very plausible for the damage costs.

We show three significant figures only to bring out the differences between these pollutants and the larger uncertainties of the secondary pollutants. But in view of the subjective and rather uncertain assumptions we had to make about the component uncertainties, we believe that it is best to simply sum up the results by saying that the geometric standard deviation of these damage costs is approximately 3. For chronic bronchitis the results are similar. For pollutants such as dioxins, As, Hg and Pb whose impacts come mostly from ingestion, we estimate, very roughly, that the geometric standard deviation is around 4.

Table 1. Uncertainty of damage cost estimates per kg of pollutant for mortality. Sample calculations of geometric standard deviation σ_g, inserting the component uncertainties σ_{gi} into Eq.6. The relative contributions of the σ_{gi} to total can be seen under $ln(\sigma_{gi})^2$.

	lognormal?	σ_{gi} **PM**	$ln(\sigma_{gi})^2$	σ_{gi} SO_2 via sulfates	$ln(\sigma_{gi})^2$	σ_{gi} NO_x via nitrates	$ln(\sigma_{gi})^2$
Exposure calculation							
Dispersion	*yes*	1.5	*0.164*	1.7	*0.282*	1.7	*0.282*
Chemical transformation	*yes*	1	*0.000*	1.2	*0.033*	1.4	*0.113*
Background emissions	*no*	1	*0.000*	1.05	*0.002*	1.15	*0.020*
Total σ_g for exposure		1.50	*0.16*	1.76	*0.32*	1.90	*0.41*
ERF							
Relative risk	*no*	1.5	*0.164*	1.5	*0.164*	1.5	*0.164*
Toxicity of PM components	*?*	1.5	*0.164*	2	*0.480*	2	*0.480*
YOLL, given relative risk	*no?*	1.3	*0.069*	1.3	*0.069*	1.3	*0.069*
Total σ_g for ERF		1.88	*0.40*	2.33	*0.71*	2.33	*0.71*
Monetary valuation							
Value of YOLL (VOLY)	*yes*	2	*0.480*	2	*0.480*	2	*0.480*
Total (Eq.6)		**2.78**	***1.04***	**3.42**	***1.51***	**3.55**	***1.61***

2.3. Placement of the Confidence Intervals

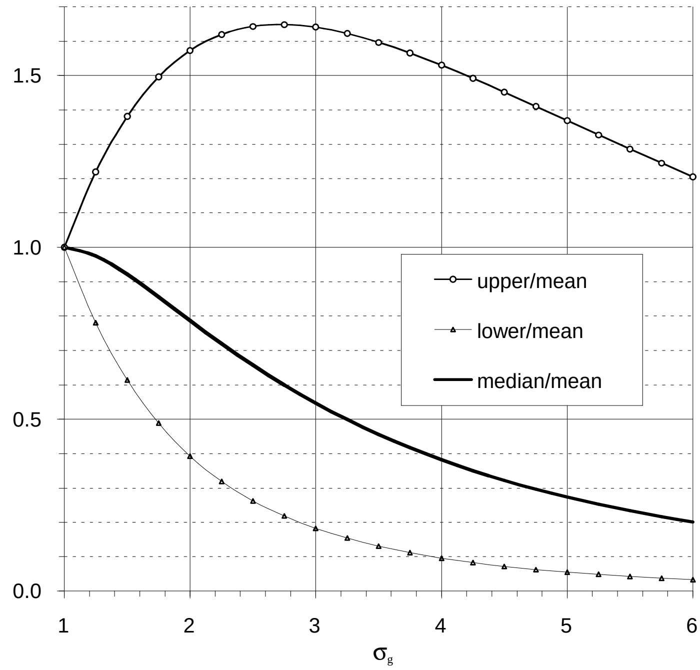
A comment is required about the placement of the confidence intervals relative to the damage cost estimates. In fact, one needs to consider whether the key parameters of the calculations have been estimated as means, medians or something else, for instance modes (= point where the probability distribution of possible parameter values has its maximum). Our informal survey of the practice of researchers in the respective disciplines leads us to the conclusion that the typical choice is the mean. Contingent valuation studies might appear to be an exception to this rule because they frequently state the median rather than the mean willingness-to-pay; however, this is done to reduce the influence of extremely high responses that are clearly unrealistic and would bias the result upward. The usual goal is to obtain the best estimate of the true population mean, as explained by Baker at al [2006].

Since the confidence intervals that have been estimated for the damage costs are symmetric around the median (=geometric mean for lognormal distribution) on a logarithmic scale, their placement relative to the quoted damage costs has to be modified. Recalling Eq.16 of Spadaro and Rabl [2007] we note that for a lognormal distribution the ratio of mean μ and median μ_g is given by

$$\mu/\mu_g = \exp\left(\frac{[\ln(\sigma_g)]^2}{2}\right) \quad . \qquad (7)$$

There is a sizeable difference between median and mean, as can be seen in Fig.1 where the ratios median/mean, upper/mean and lower/mean are plotted as function of the geometric standard deviation σ_g for a lognormal distribution. For example, with $\sigma_g = 3$ the mean/median ratio μ/μ_g is 1.83.

Fig.1. Median (μ_g), upper bound ($\mu_g \times \sigma_g$) and lower bound (μ_g/σ_g), all divided by the mean μ, as function of the geometric standard deviation σ_g for a lognormal distribution.



To see what this implies for the placement of the upper and lower bounds, let us consider the numbers for PM damage cost according to the UWM of Eq.1. Table 2 shows the results for the placement of the confidence intervals. We take σ_g of the depletion velocity v_{dep} as the total for atmospheric modeling, namely the combination of the $\sigma_{g,i}$ for dispersion and chemical transformation, as per Table 1. For PM the ratio lower bound/mean is 0.59/2.78=0.21 and the upper bound/mean is 0.59 * 2.78 = 1.65.

Table 2. Placement of the 68% confidence intervals [Low, High] for UWM.
Mean = μ, median = μ_g. The equations used for this can be found in Spadaro and Rabl [2007].

	PM	SO_2 via sulfates	NO_x via nitrates
μ/μ_g for exposure (v_{dep})	1.09	1.17	1.23
μ/μ_g for CRF	1.22	1.43	1.43
μ/μ_g for monetary value p	1.27	1.27	1.27
μ/μ_g for UWM	1.68	2.13	2.23
μ_g/μ for UWM	0.59	0.47	0.45
Low/μ_g = 1/σ_g for UWM	0.36	0.29	0.28
High/μ_g = σ_g for UWM	2.78	3.42	3.55
Low/μ for UWM	**0.21**	**0.14**	**0.13**
High/μ for UWM	**1.65**	**1.61**	**1.59**

Since the estimation of uncertainties is extremely uncertain, we believe that it would be appropriate to cite just a single set of results for air pollutants such as PM, SO_2 and NO_x that act via inhalation. For policy applications typical uncertainties are more instructive than detailed values of geometric standard deviation for each source and each impact. For inhalation of most air pollutants we suggest a typical geometric standard deviation of 3 and the ratios median/mean ≈ 0.5, low/mean ≈ 0.2 and high/mean ≈ 1.6. For toxic metals we estimate, very roughly, that σ_g might be around 4; the corresponding ratios are median/mean ≈ 0.38, low/mean ≈ 0.1 and high/mean ≈ 1.5.

2.4. Sum over Endpoints, Impact Categories or Pollutants

We have also developed a simple method for estimating the uncertainty of the sum of damage costs for a set of different impact types or pollutants, and we have verified its usefulness by means of detailed Monte Carlo calculations. We have verified with Monte Carlo calculations for a large number of realistic cases that the sum of lognormally distributed damage costs is, to an excellent approximation, also lognormal. Then we have derived two analytical estimates for the geometric standard deviation of the sum, one an over- the other an underestimation. Finally we have found that the average of these two estimates turns out to be remarkably close to the correct answer. The details can be found in Spadaro and Rabl [2007].

As a general observation we note that whereas the absolute error of a sum is larger than the absolute errors of the summands, the relative error is smaller than the largest relative error of the summands: in the cases we have examined for air pollution, the relative error of the sum can even be significantly smaller. For the cost per kWh, for most energy systems, there are contributions from PM_{10}, NO_x and SO_2, as well as the greenhouse gases. For the latter the uncertainty is much larger than for the former; it is difficult to estimate but might correspond to a geometric standard deviation of 5. Thus the geometric standard deviation of the cost per kWh is intermediate between 3 and the one of the greenhouse gases, and in most cases it is closer to the former than the latter.

3. Uncertainties for Greenhouse Gases

As explained at the end of the Introduction, we have not been able to implement the detailed approach of Section 2 for climate change. As an alternative we have looked at the range of damage cost estimates in the literature, in particular the recent review by DEFRA [2004, 2005] in the UK and the review of Tol [2005], see Section 3.1. We have also developed a simplified model of damage and abatement costs, in order to carry out a sensitivity study of the optimal emissions level; that is described in Section 3.2.

3.1. Review of the Literature

A number of integrated assessment models have been developed to assess climate change damage costs and their evolution over time. Some are based directly on the impact of ΔT on Gross Domestic Product (GDP) or Gross World Product (GWP).

Others, in particular FUND (Tol, 1995) and PAGE (Plambeck and Hope, 1996), attempt to simulate climatic impacts in more detail, e.g. according to the categories or economic sectors to which they apply. One of the pioneering models in this field was DICE (Nordhaus, 1991 and 1994). DICE optimizes the trade-off between the costs of climate change and the costs of restricting CO_2 emissions. The damage cost simulation of DICE is based on the assumption that a 3 ˚C warming induces a 0.25 % loss of GDP in the USA, based on estimates of market damages such as crop loss, forestry impact, and shoreline erosion. This value is raised to 1 % to account for all probable damages, especially non-market ones that are generally hard to quantify. In order to render DICE applicable globally the relative loss is further increased to 1.3 % of GWP, as many less developed countries are more dependent on e.g. agriculture and have as such a more limited ability to adapt to the effects of climate change. Furthermore, Nordhaus recognizes that for temperature rises higher than 3 ˚C disproportionally large damages are likely to result, so that a quadratic function is then a more appropriate relation. Most of the subsequent climate-economy models have adopted a similar climate change damage cost formulation. Since it is broadly accepted that this formulation is among the most significant determinants of the outcomes produced with these models, the present article focuses on the sensitivity of climate policy making to the representation of, and uncertainties in, climate change damage costs.

Another widely used climate policy assessment model is MERGE, a multi-region Ramsey-Solow optimal growth model including greenhouse gas emissions and a global climate module (Manne and Richels, 2004). It can be operated in a cost-benefit mode, in which a time path is chosen for emissions that maximizes the integrated discounted utility of consumption, after making allowance for the disutility associated with climate change. The assumptions in MERGE with regards to the damages generated by climate change are acknowledged to be highly speculative. At present, it is challenging to render them more accurate given the rudimentary state-of-the-art of climate change damage assessment. They remain for the moment based on informed judgment and inspection of the literature produced so far. Whereas MERGE includes both market and non-market damages, it focuses on the latter, as they are considered the largest. In particular, market and non-market damages are assumed to be linear and non-linear with temperature increases, respectively, and follow the type of assumptions made in DICE (Nordhaus, 1994). Thus, the loss resulting from climate change, possibly even climatic catastrophe, is supposed to increase disproportionally (in this case again quadratically) if mankind passes beyond an average atmospheric temperature increase of a few ˚C. While different numerical assumptions are made for different regions in the world, Manne and Richels (2004) presume that for a ΔT of 2.5 ˚C an economic loss of 2 % of GDP is incurred in high-income countries (in other words, the willingness-to-pay to avoid such a temperature increase is 2 % of GDP).[1] This fraction of GDP, which can be interpreted as a reduction in welfare, is lost for conventional consumption by households and government. At the basis of simulations performed with models like FUND and PAGE, and of the other modeling exercises referred to below (Cline, 1992; Fankhauser, 1995; Titus, 1992), are similar damage cost assumptions as in DICE and

[1] For low-income countries, like China and India, the 'hockey-stick'-parameter they use in MERGE is smaller than 1. This means that at a per capita annual income between $5,000 and $50,000 a region is only willing to pay 1 % of GDP to avoid a 2.5 ˚C temperature rise, and at $5,000 or below basically nothing. At $50,000 or above the 2 % of GDP willingness-to-pay applies.

MERGE, in some cases detailed per sector and/or region.[2] Still, the parameter choices and corresponding specific quantifications of damage costs, and thus the numeric way the quadratic temperature dependence is introduced in these models, often vary substantially, as shown in the next section.

Two recent studies have produced an overview of an important part of the climate change damage literature and made a comparison of two modeling exercises determining the marginal damage cost of CO_2 (DEFRA, 2004 and 2005). They report calculations of the 'social cost of carbon', that is, the derivative dC_{dam}/dE of the total damage cost C_{dam} with respect to the emissions level E. Fig.2 shows the main results from these analyses: 'central guidance' values for dC_{dam}/dE (thick middle curve) with one set of lower and upper central estimates (dashed curves) and a set of lower and upper bound estimates (thin outer curves), specified for six points in time until 2050. We have transformed the original data into units of €/t_{CO2}, using an exchange rate of 1.5 €/£. Also, in view of the remainder of our analysis, we have solely used values for dC_{dam}/dE, rather than a merger between data for dC_{dam}/dE and marginal abatement cost as reported by DEFRA (2004) and interpreted as carbon shadow prices. Otherwise, the data behind Fig.2 follow the recommendations of DEFRA (2004).

The middle curve represents the average of the time-dependent means as calculated by FUND and PAGE, two models commonly applied in this field. Both are integrated assessment models designed to jointly simulate economic growth, CO_2 emissions, and climate change impacts.[3] These values are recognized to not cover all climate change damage costs, as they exclude, for example, major climatic events, socially contingent effects and many of the non-market impacts. For the economic assessment of catastrophic climate change impacts CBA as used in these models and the one we present below has strong limitations, so that it is preferable to apply other types of analysis instead (see e.g. Weitzman, 2007 and Yohe, 1996). Damage cost estimates strongly depend on the time discounting method, assumptions regarding equity weighting, and the supposed risk aversion or willingness-to-pay to avoid damages. The dC_{dam}/dE numbers generated by FUND and PAGE are based on relatively low and declining discount rates, and include moderate global assumptions on equity weighting and risk aversion. These subjects remain matters of continued scientific and political debate.

The other four curves indicate the uncertainty range of the central graph. The dashed curves directly below and above the middle one represent lower and upper estimates for the central guidance of dC_{dam}/dE. For the lower central curve a value of 14 €/t_{CO2} is taken in 2000, as proposed by DEFRA (2005). For its evolution over time it is assumed that the ratio (central / lower central) remains fixed. The lower central estimate of 14 €/t_{CO2} is considered reasonable for a global decision context committed to reduce the threat of dangerous climate change under relatively low time discounting and equity weighting and a modest level of risk aversion to extreme damages. The 95% confidence level numbers calculated by PAGE are used for the upper central curve, to

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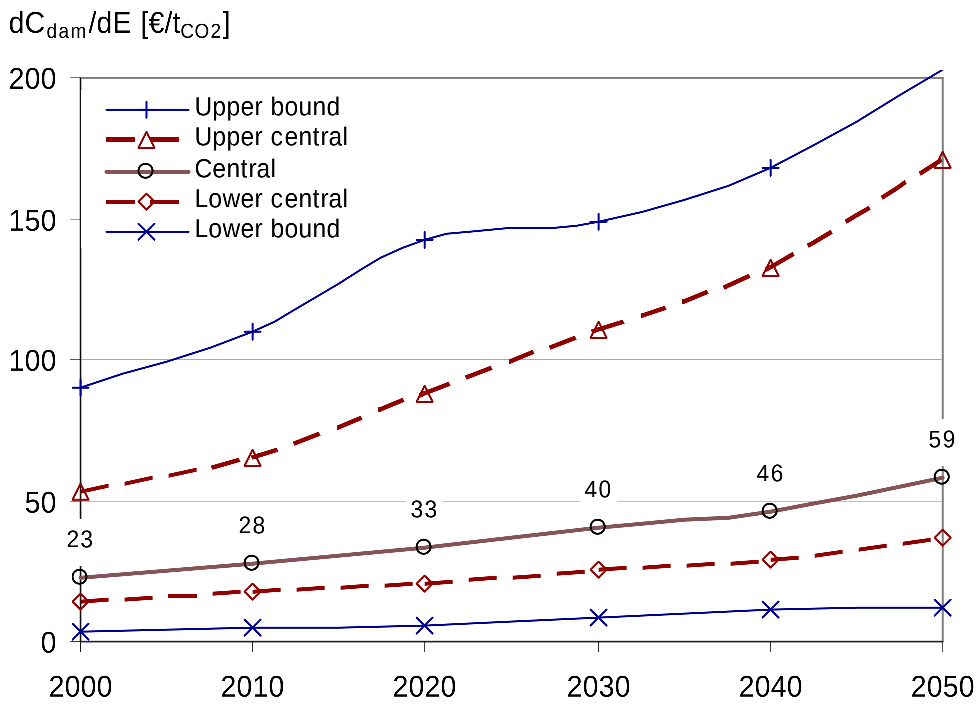
[2] Differences sometimes exist, however, in assumptions on the tolerable temperature rise, defined as the ΔT below which no climate change damage is expected. While most models suppose a tolerable temperature of 0 ˚C, Manne and Richels (2004) assume it to be the temperature level in 2000 (which was about 0.7 ˚C higher than the average pre-industrial value) and Plambeck and Hope (1996) 2 ˚C.

[3] For the original descriptions of these models, see for example Tol (1995) and Plambeck and Hope (1996), respectively.

express that high values for dC_{dam}/dE may well occur, as a result of e.g. unpredictable but possibly large social contingencies. The thin outer bound curves reflect an even larger cost range uncertainty. The lower bound is the 5% CL estimate of PAGE, and the upper bound the average between the 95% CL simulations of FUND and PAGE. Even though the range thus obtained is enormous, it still does not fully capture all published estimates. Negative dC_{dam}/dE values have been reported (implying net climate change benefits rather than costs) as well as values a couple of times higher than the top dC_{dam}/dE value of 200 €/t_{CO2} in 2050.

Fig.2. The marginal damage cost of CO_2, dC_{dam}/dE, expressed in €/t_{CO2}, versus year of emission. Data based on DEFRA (2004 and 2005).[4]



In addition to presenting the future evolution of marginal damage costs of Fig.2 as derived with FUND and PAGE, DEFRA (2004) reports as estimate for dC_{dam}/dE applicable today a mean value of 33 €/t_{CO2}, with 5% and 95% values of respectively -4 €/t_{CO2} and 123 €/t_{CO2}, based on a summary of results found through a literature review. The estimates for 2000 reported in Fig.2 fall well within this range. DEFRA (2004) recommends to use the curves of Fig.2 in a multi-level-approach. The central range is intended for most day-to-day policy appraisal (like in relatively minor short-term infrastructure construction projects or local environmental impact assessments) and the full range for major long-term purposes (such as the national planning of an energy security strategy) or uncertainty analysis in a global cost-benefit framework (as in this paper).

[4] Unlike reported in DEFRA (2004), we have not rounded and fitted C_{dam} values so as to obtain 'overall-trend' distributions allowing the use of fixed rates of increase.

Tol (2005) also reviews a large number of climate change impact studies, and combines over 100 estimates for the marginal damage cost of CO_2 to form an overall probability density function. The uncertainty proves to be strongly right-skewed, with a median of \$3.8/t_{CO2}, a mean of \$25.4/t_{CO2}, and a 95% CL of \$95/t_{CO2}. According to Tol (2005), under standard assumptions of time discounting, equity weighting, and risk aversion, the marginal damage cost is unlikely to exceed \$14/t_{CO2}, and is probably even smaller. This value is significantly lower than the \$85/t_{CO2} reported by the widely publicized Stern Review (Stern *et al*., 2006). The discrepancy can to a large extent be explained by the very low value of the social discount rate employed in their review, in comparison to the discounting conventionally used (Dasgupta, 2006 and Nordhaus, 2006).

The highly skewed distribution of damage cost estimates in the literature (DEFRA, 2004 and Tol, 2005) is fairly consistent with a lognormal function, even though a few studies claim negative damages. Indeed, global climate change probably produces both winners and losers, at least at moderate temperature increases, but we do not believe that the net world-wide damage cost could be negative for any increase of the atmospheric CO_2 concentration. As representation of the estimates found in the literature we therefore take a lognormal distribution, and choose for its parameters a median μ_g = \$3.8/t_{CO2} and upper limit $\mu_g\ \sigma_g^2$ = \$95/t_{CO2}, that is, $\sigma_g = 5$.[5] The numbers are reasonably consistent with those of DEFRA. In view of the limitations of currently available studies – notably the fact that especially some of the most troubling potential impacts, such as a change in the thermohaline circulation, rapid non-linear ice-sheet disintegration, or methane release from permafrost melting, have not yet adequately or hardly at all been taken into account – we realize that the uncertainty range may well be larger than $\sigma_g = 5$.

3.2. A model for the damage cost as function of emissions

Most of the integrated assessment modeling studies on energy, climate change, and the economy, use a damage cost function with the shape:

$$C_{dam} = \rho(\Delta T_{stab})^{\theta}, \tag{8}$$

in which C_{dam} is the damage cost expressed as fractional loss of GWP, ΔT_{stab} the global average temperature change with respect to the pre-industrial atmospheric temperature in stabilized conditions, i.e. when equilibrium is reached of the climate system, and ρ and θ coefficients characterizing the shape of the damage function. GWP is a commonly employed measure for the size of the global economy that by definition only covers tangible markets. It is therefore incomplete, as not all market activity always enters national accounts, especially in developing economies, and it does certainly not reflect non-market activity. Still, percentage losses of GWP are often used to express both market and non-market damage costs, as GWP constitutes a convenient unit of measure.

5 This median is the one reported in the review by Tol (2005) and the upper limit the 95% confidence interval in this reference.

Central to our analysis is the definition of ΔT_{stab} as the stabilized global average temperature change obtained after a specified emission profile leads to new equilibrium values of the atmospheric CO_2 concentration and the corresponding increase in atmospheric temperature. Time lags exist both between the CO_2 emissions level and the stabilized CO_2 concentration, and between this new atmospheric CO_2 concentration and ΔT_{stab}, typically in each case of at least several decades up to a century.

Uncertainties about the parameters ρ and (abound. The function of Eq.8 is usually assumed to be quadratic, so that (is 2. Roughgarden and Schneider (1999) investigate values of (other than 2 (both 1<(<2 and (>2) on the basis of a set of expert views. They conclude, however, that a quadratic damage function is most plausible: while (=2 is not a necessity – the damage function may e.g. be somewhere in between linear and quadratic – differences of opinion on climate damage costs show up primarily in the coefficient ρ of the damage function, rather than in its exponent. Roughgarden and Schneider (1999) argue that allowing for views from experts of different scientific disciplines – who have differing opinions on especially the likelihood of extreme climate events – implies variations of ρ by as much as an order of magnitude, but in most of the literature one finds values for ρ that typically lie between 0.001 and 0.004. Table 3 summarizes the values of the coefficient ρ as obtained from a survey of some of the most widely used integrated assessment models of climate change.

Formulated slightly differently, Manne and Richels (2004) assume in MERGE the relation:

$$C_{dam} = \left(\frac{\Delta T_{stab}}{\Delta T_{cat}} \right)^2 , \qquad (9)$$

in which ΔT_{cat} is the catastrophic temperature change at which all economic activity, hence the entire GWP, is supposed to be wiped out. Combining Eqs.8 and 9 one finds the ΔT_{cat} implicit in the models behind the references listed in Table 3.

Table 3. Parameter values for ρ and corresponding ΔT_{cat} as assumed in several widely employed integrated assessment models of climate change.

Source	ρ	ΔT_{cat} **(˚C)**
Cline (1992)	0.0023	20.7
Fankhauser (1995)	0.0028	19.0
Manne and Richels (2004)	0.0032	17.7
Nordhaus (1991, 1994)	0.0015	26.0
Plambeck and Hope (1996)	0.0028	19.0
Titus (1992)	0.0021	21.9
Tol (1995)	0.0032	17.7

N.B. Most of these authors report damages relative to GDP in the USA for one temperature increase level only, typically as associated with a doubling of the atmospheric CO_2 concentration. Roughgarden and

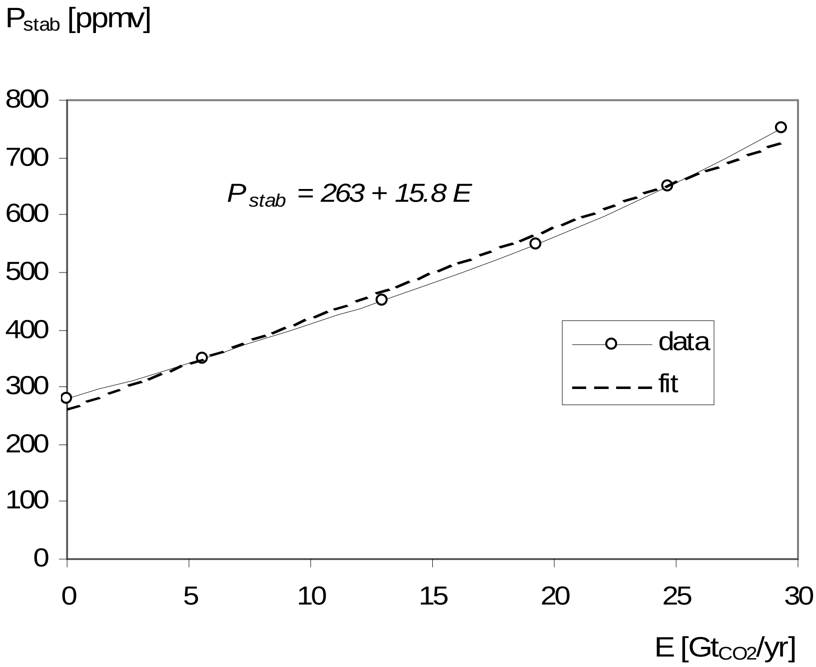
Schneider (1999) apply Nordhaus' assumptions to the figures adopted by them in order to obtain expressions for the damage function of Eq.8 consistent with DICE.

Since for our analysis we need to express C_{dam} as function of E, we first relate $\otimes T_{stab}$ to the atmospheric CO_2 concentration P by recalling that a roughly logarithmic relation exists between this concentration and global temperature increase (Houghton *et al.*, 1996). More precisely, the stabilization level of atmospheric CO_2 concentration, P_{stab}, can be related to $\otimes T_{stab}$ by (see Caldeira *et al.*, 2003):

$$\frac{P_{stab}}{P_{280}} = 2^{(\frac{\Delta T_{stab}}{\Delta T_{2X}})}, \qquad (10)$$

in which P_{280} is the pre-industrial CO_2 concentration of about 280 ppmv, and $\otimes T_{2X}$ the climate sensitivity, defined as the temperature change $\otimes T_{stab}$ resulting from a doubling of the atmospheric CO_2 concentration. Hence, a stabilization concentration target for atmospheric CO_2 increases exponentially with the ratio of the stabilization temperature change and the climate sensitivity. As before, neither $\otimes T_{stab}$ nor $\otimes T_{2X}$ are instantaneous temperature changes, but global mean surface temperature changes attained if CO_2 concentrations are held constant long enough to reach stable average climate conditions. $\otimes T_{2X}$ is thought to lie in a range of about 1.5 to 4.5 ˚C. Recent empirical studies (based on e.g. ice core measurements) indicate that there is significant likelihood that $\otimes T_{2X}$ lies above this canonical range (see for a recent overview of possible values of $\otimes T_{2X}$ and implications, for example, van der Zwaan and Gerlagh, 2006). A most likely value of 3 ˚C for $\otimes T_{2X}$ may thus still be considered conservative. In order to derive a simple time-independent relationship between P_{stab} and E, we employ the curves of Figure 1 in Wigley *et al.* (1996) representing a set of time-dependent emission profiles for the period 1990 – 2300 calculated for different values of P_{stab}. We take for each value of P_{stab} the corresponding annual emission level E averaged over this time frame. The data and a linear fit are shown in Fig.3.

Fig.3. Time-independent relationship between P_{stab} and E based on the data of Wigley et al. (1996), obtained by setting E equal to the annual average of their 1990-2300 emission profiles. The data points show E for P_{stab} = 280, 350, 450, 550, 650, and 750 ppmv. The straight line is our linear regression.



Let E_s be the current emissions value of about 25.7 Gt_{CO2}/yr. By using P_{280} and E_s as reference levels we represent the linear fit of Fig.3 in dimensionless form:[6]

$$\frac{P_{stab}}{P_{280}} = \varepsilon + \delta \frac{E}{E_s} \quad \text{with } \delta = 1.45 \text{ and } \varepsilon = 0.94. \tag{11}$$

Combining Eqs.8, 10 and 11, and assuming (= 2, we obtain this model of the damage cost.

$$C_{dam}(E) = \rho \left[\Delta T_{2X} \ln(\varepsilon + \delta \frac{E}{E_s}) / \ln 2 \right]^2 . \tag{12}$$

The marginal damage cost is of course the derivative with respect to the emissions E

$$\frac{dC_{dam}}{dE} = \frac{2}{\ln 2^2} \rho \Delta T_{2X}^2 \frac{\ln(\varepsilon + \delta E / E_s)}{\varepsilon + \delta E / E_s} \frac{\delta}{E_s} . \tag{13}$$

We have also carried out a Monte Carlo analysis, using the CrystalBall software. For the distributions of the parameters of the damage cost curve we assume normal distributions with the following means and standard deviations:

[6] This choice for E_s is arbitrary and has no effect on our final results.

δ: Normal (1.45, 0.10)
ε: Normal (0.94, 0.05)
ρ: Normal (0.0020, 0.0005)
ΔT_{2X}: Normal (3, 1.2)

The resulting distribution of marginal damage costs is highly skewed, although fairly different from lognormal; it has a geometric standard deviation of about 3.5. It is easy to get higher or lower values for the geometric standard deviation by making different assumptions on the distributions of the parameters. On balance we feel that a geometric standard deviation in the range of 4 to 5 seems reasonable for current estimates of the marginal damage cost of climate change.

3.3. A model for the Abatement cost as function of emissions

Like in Rabl *et al*. (2005), we assume that the marginal CO_2 abatement cost takes the functional form:

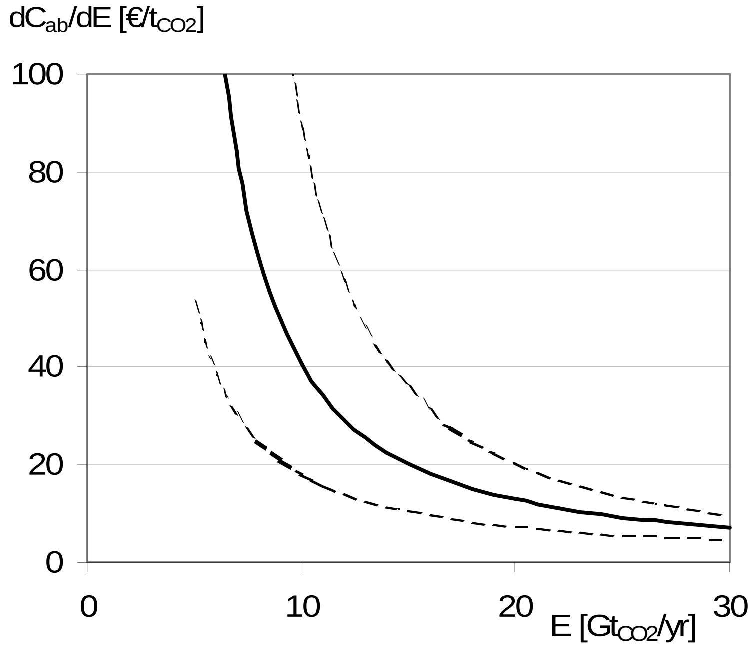
$$\frac{dC_{ab}}{d(-E)} = \frac{1}{G}\alpha\left(\frac{E-\beta}{E_s}\right)^{\gamma}, \tag{14}$$

in which C_{ab} is the globally aggregated abatement cost as fraction of GWP (abridged to G in this formula), E and E_s (both in Gt_{CO2}/yr) are as before, and α (in €/tCO_2), β (in Gt_{CO2}/yr) and γ are coefficients characterizing the non-linear convex form of the abatement cost function. The factor G is added to the relation used in Rabl *et al*. (2005) in order to stay consistent with the way damage costs are expressed in Eq.8 and because in the present analysis we find it convenient to express all costs as fraction of GWP. For GWP we adopt today's value of approximately 50 trillion €/yr. We choose the signs in Eq.14 so that dC_{ab} is positive for a reduction of E. Also α is assumed to be positive. The parameters α, β, and γ may be determined by least squares regression if cost data are available as a function of the abatement level. Alternatively, they may be estimated on the basis of energy technology assessments or energy systems modeling. Here we choose the marginal abatement cost curves depicted in Fig.4, based on an evaluation of published integrated assessment modeling results (see notably Goulder and Mathai, 2000; Goulder and Schneider, 1999; van der Zwaan *et al*., 2002; Yohe, 1996).[7] Fig.4 contrasts with near-term abatement cost curves obtained through detailed engineering energy technology analyses, like with the GAINS model up to 2020 (Klaassen *et al*., 2005). Over such a short time frame the potential for deep reductions is limited or exceedingly costly, because many of the technological options available require long installation lead times or have costs that are unacceptably high at the present time. In the long run, however, major cost reductions are to be expected as a result of technological

[7] These references typically report shadow carbon prices, which we associate with the efforts needed to achieve carbon emission reductions or, alternatively, carbon abatement costs.

progress and learning-by-doing. Thus, our abatement cost assumptions concern the long run and should not be interpreted as realistic short-term policy goals.[8]

Fig.4. Our choice for the marginal abatement cost curve (solid line) and lower and upper bounds (dashed lines). The coefficients are α=7.5, β=3, and γ=–1.3 for the central curve, α=5, β=2, and γ=–1.1 for the lower limit, and α=10, β=4, and γ=–1.5 for the upper limit.



The cost of reducing CO_2 emissions from starting point E_s to level E is the integral of the marginal abatement cost equation of Eq.14:

$$C_{ab}(E) = \frac{1}{G}\frac{\alpha E_s}{\gamma+1}\left[\left(\frac{E_s-\beta}{E_s}\right)^{\gamma+1} - \left(\frac{E-\beta}{E_s}\right)^{\gamma+1}\right] \quad \text{for } \gamma \neq -1. \tag{15}$$

We have also carried out a Monte Carlo analysis, using the CrystalBall software. For the distributions of the parameters of the abatement cost curve we assume normal distributions with the following means and standard deviations:

α: Normal (7.5, 1.0)
β: Normal (3, 0.5)
γ: Normal (-1.3, 0.1)

The resulting distribution of marginal abatement costs is approximately normal, except when the emission level drops below about 8 Gt_{CO2}/yr. The ratio of standard deviation and damage cost is about 0.15 at E = 25 Gt_{CO2}/yr, increasing to about 0.25 at E = 10 Gt_{CO2}/yr.

[8] Figure 3 shows marginal abatement costs only down to E = 5 Gt_{CO2}/yr, as below this reduction level they become much higher than marginal damage costs, such that effectively no mitigation takes place. Also, below this abatement level the cost uncertainties are too extreme to be of real significance.

4. Effect of the uncertainties

4.1. Different levels of internalization

Whereas there are many different policy instruments that can be used to reduce the emission of a pollutant, pollution taxes and tradable permits are of special interest because they can achieve greater economic efficiency, being based on the market rather than command-control. They are increasingly used, especially for greenhouse gases.

Internalization of the damage costs of a pollutant can be interpreted in two different ways:
i) the polluter reduces the emission to the social optimum;
ii) the polluter reduces the emission to the social optimum and pays an amount equal to the residual damage.
The first level is achieved by tradable permits that are given away free or by an emission limit equal to the social optimum. The second level is achieved by a pollution tax (equal to the marginal damage) and also by tradable permits that are auctioned by the government. In the spirit of internalization the second level is appropriate if the payment for the residual damage is used to compensate the victims. In practice the victims of air pollution and their individual losses cannot be identified with sufficient precision to bring about any reasonably correct compensation. Nonetheless a pollution tax may be desirable as replacement of other taxes; in particular compared to income tax it entails no direct disincentive to earn more money. In practice different countries apply different policy instruments, and both internalization levels can be found.

In the case of an emission limit the polluter pays only the abatement cost necessary to respect the limit. With tradable permits that are given away free the detailed monetary transfers depend on the allocation of the permits, but in any case an equilibrium price of the permits is reached that is equal to the median abatement cost: those whose abatement cost is less sell their permits, the others buy them. On average the polluters pay only the abatement cost, just as for emission limits (such tradable permits are in fact an emission limit that is imposed on the polluters as a group). In the case of a pollution tax or permits that are auctioned, the polluters pay both the abatement cost and the cost of the residual damage. The difference in cost to the polluter between these two internalization levels is the cost of the residual damage. It can be very large.

Thus the monetary transfers and the effects on the economy can be much larger in the case of a tax or auctioned permits, and so are the consequences of errors in the estimation of the external costs. To compare the consequences of errors we consider global warming and assume the model of van der Zwaan and Rabl [2007] for the total world wide the damage cost C_{dam}. The marginal damage cost is equal to

$$\frac{dC_{dam}}{dE} = \frac{2}{\ln 2^2} \rho \Delta T_{2X}^2 \frac{\ln(\varepsilon + \delta E / E_s)}{\varepsilon + \delta E / E_s} \frac{\delta}{E_s} \qquad (16)$$

where E = world wide emissions, and E_s is the current emission level of about 25.7 Gt_{CO2}/yr. The other parameters, δ, ε, ρ and ΔT_{2x}, are parameters of the model. At the current emission level the marginal damage cost is 77 €/t_{CO2}, decreasing only slightly as E is reduced: for instance at E = 20 Gt_{CO2}/yr it is 74 €/t_{CO2}. On the other hand the damage cost estimate of DEFRA [2004] is 28 €/t_{CO2} for emissions in 2010, increasing to 59 €/t_{CO2} for emissions in 2050.

The total CO_2 emissions in the EU are currently about 3.5 Gt_{CO2}/yr in the EU-15. If the marginal damage cost is 28 €/t_{CO2}, extra cost to the polluters for internalization level (ii) is 28*3.5*10^9 €/yr = 98 billion €/yr, about 1% of the GDP of the EU-15. Since damage cost are very uncertain, maybe by a factor of 5, this figure could be in the range of 0.2% to 5%. In any case the transfer payments under internalization level (ii) are very large. The consequences for the economy should be evaluated carefully, but such an analysis is complicated and clearly beyond the scope of the present project.

The damage costs for NO_x and SO_2 vary somewhat with emission site. Typical values in the EU-15 are approximately 3500 €/t, for each of these pollutants. Current SO_2 emissions are 5*10^6 t_{SO2}/yr in the EU-15. The extra cost to the polluters for internalization level (ii) is 5*3500*10^6 €/yr = 17.5 billion €/yr, about 0.2% of the GDP of the EU-15. Current NO_x emissions are 9.3*10^6 t_{NOx}/yr in the EU-15. The extra cost to the polluters is 9.3*3500*10^6 €/yr = 32.6 billion €/yr, about 0.3% of the GDP of the EU-15. The numbers for NO_x and SO_2 are much smaller than for CO_2 but not negligible. These results are summarized in Table 4.

Table 4. Difference of cost to polluters between internalization levels (i) and (ii).

Pollutant	**Marginal damage cost**	**Extra cost for level (ii)**	
		billion €/yr	**% of GDP of EU-15**
CO_2	28 €/t_{CO2}	98	about 1%
NO_x	3500 €/t_{NOx}	33	about 0.3%
SO_2	3500 €/t_{SO2}	18	about 0.2%

4.2. Effect on Policy Choices

Some people question the usefulness of damage cost estimates because of their large uncertainties. Here we address that objection by asking "how large is the social cost penalty if one makes the wrong choice because of uncertainties in the cost or benefit estimates?" We examine one the most important and general policy choices, namely the choice of national or global emission ceilings. For NO_x and SO_2 we have already published the results before this project [Rabl, Spadaro and van der Zwaan 2005] (PM_{10} was excluded from that paper because as a primary pollutant its damage cost varies so much with emission site that a much more detailed and complicated analysis would be required for which we did not have the necessary data).

The methodology in that paper was to use curve fits to data of abatement cost C_{ab} as function of emission level E. For the marginal damage cost $D = dC_{dam}/dE$ we assumed a constant value (but for NO_x and SO_2 different in different countries). The analysis was done at the country level for NO_x and SO_2, at the global level for CO_2. We determined the optimal emission level $E_{o,est}$ (i.e. the optimal emission ceiling) by minimizing the total cost $C_{tot} = C_{dam} + C_{ab}$. This is the estimated optimum, based on the available estimate of the marginal damage cost, D_{est}. Since the true optimum $E_{o,true}$, corresponding to the true damage cost D_{true}, is different from the estimated one, the realized total cost will be higher than the true minimum. The social cost penalty ΔC is the difference between the realized total cost $C_{tot}(E_{o,est})$ and the true minimum $C_{tot}(E_{o,true})$.

$$\Delta C = C_{tot}(E_{o,est}) - C_{tot}(E_{o,true}) \tag{17}$$

We also consider the cost penalty ratio

$$R = C_{tot}(E_{o,est})/C_{tot}(E_{o,true}) \tag{18}$$

as function of $x = D_{true}/D_{est}$, both numerator and denominator being evaluated with the true damage cost D_{true}.

We found a remarkable insensitivity to uncertainties. For NO_x and SO_2 an error by a factor of three increases the total social cost by at most 20%, and in most cases much less, i.e. R is less than 1.2. That paper also looked at dioxins and CO_2, pollutants for which the uncertainties of the damage cost are even larger than in the case of NO_x and SO_2. Nonetheless we found similar insensitivity to uncertainties even for these pollutants.

For the present project we have redone the analysis for CO_2, using the more realistic and detailed models of Sections 3.2 and 3.3. The optimal emission level E_o is found by minimizing $C_{tot}(E)$, hence by setting the sum of the marginal abatement cost and the marginal damage cost equal to zero, so that:

$$-\frac{\alpha}{G}\left(\frac{E-\beta}{E_s}\right)^{\gamma}+\frac{2}{\ln 2^2}\rho\Delta T_{2X}^2\frac{\ln(\varepsilon+\delta E/E_s)}{\varepsilon+\delta E/E_s}\frac{\delta}{E_s}=0 \quad \text{at } E = E_o. \tag{19}$$

Unlike the case with linear damage costs, there is no analytical solution to this relation. We thus use the FindRoot function of Mathematica® to obtain a numerical solution.[9] Table 5 lists the central values, $p_{central}$, of all parameters of the optimization problem, as well as their ranges, $[p_{min}, p_{max}]$, considered for the uncertainty analysis. For the central values of these parameters the optimal emission level is found to be E_o = 8.7 Gt_{CO2}/yr, about one third of the current emission level E_s. The marginal damage and abatement costs at E_s are about 77 €/t_{CO2} and 9 €/t_{CO2}, respectively, while at the optimum they are equal to approximately 54 €/t_{CO2}. Fig.5 shows the abatement, damage, and total costs expressed as percentage loss of GWP as function of emission level E. At today's emissions the damage cost clearly dominates, which highlights the need for major reductions.

[9] All numerical results presented in this paper have been calculated with Mathematica.

Table 5. Central values and ranges of parameters p for the damage and abatement costs in the optimization problem.

Parameters *p*	p_{min}	$p_{central}$	p_{max}
Damage cost $C_{dam}(E)=\rho\left[\Delta T_{2X}\ln(\varepsilon+\delta\frac{E}{E_s})/\ln 2\right]^2$			
δ	1.15	1.45	1.75
ε	0.74	0.94	1.14
ρ	0.0005	0.0020	0.0035
ΔT_{2X} [ºC]	1	3	5
Abatement cost $C_{ab}(E)=\frac{1}{G}\frac{\alpha E_s}{\gamma+1}\left[\left(\frac{E_s-\beta}{E_s}\right)^{\gamma+1}-\left(\frac{E-\beta}{E_s}\right)^{\gamma+1}\right]$			
α [€/t_{CO2}]	5.0	7.5	10
β [Gt_{CO2}/yr]	2	3	4
γ	-1.5	-1.3	-1.1

Fig.5. Damage, abatement, and total costs expressed as percentage loss of GWP as function of emissions level E.

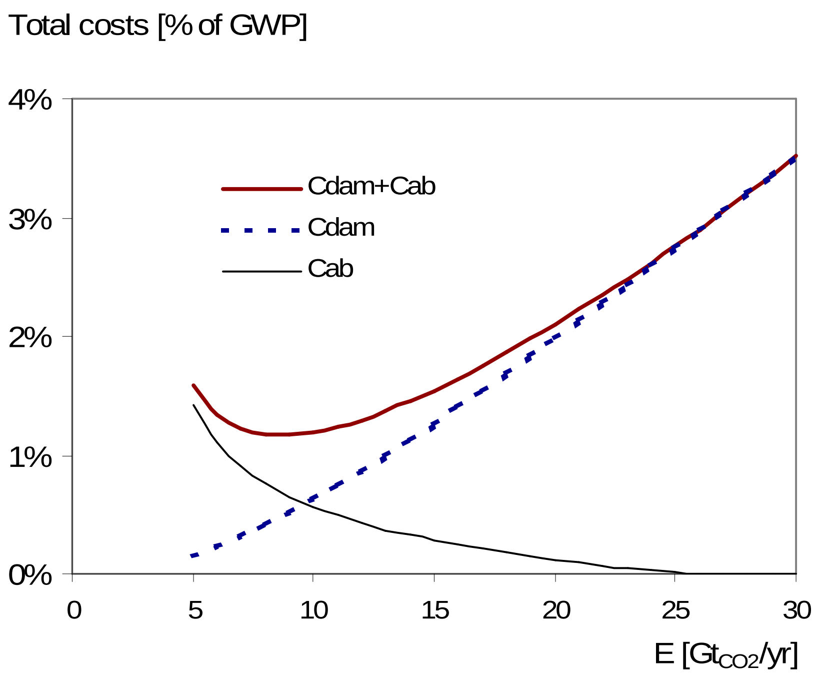
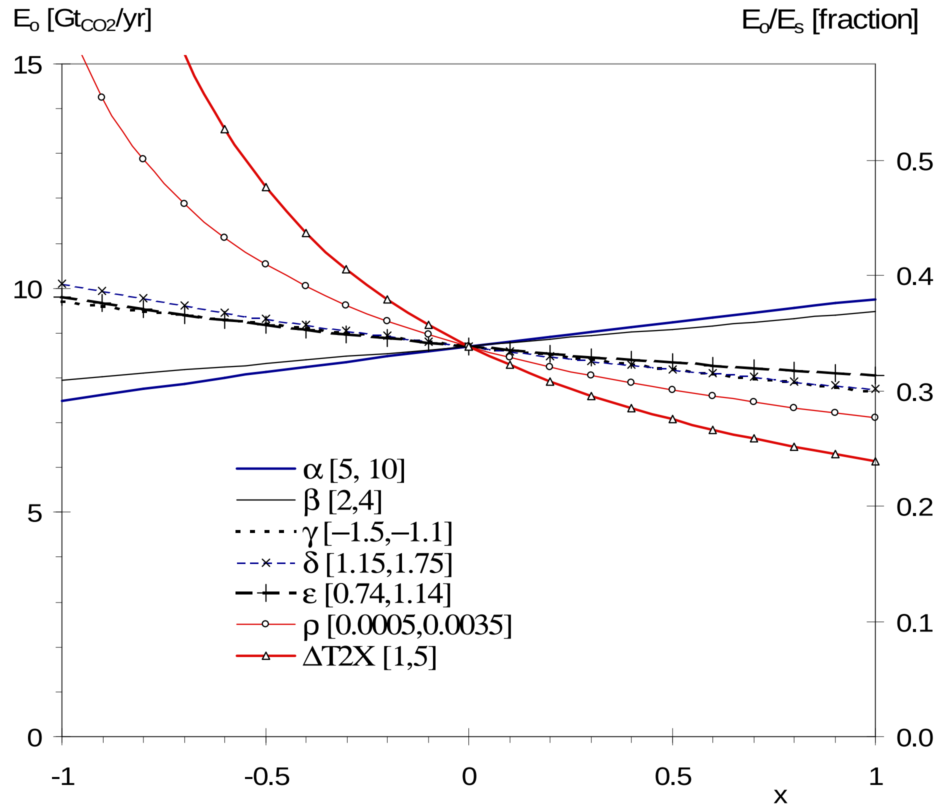


Fig.6 shows our results regarding the dependence of the optimal emissions level E_o on the parameter assumptions of our optimization problem. Since G and E_s are known with sufficient precision – their uncertainties are essentially negligible – they do not need to be subjected to a sensitivity analysis. We vary each parameter p over the range [p_{min}, p_{max}] as listed in Table 5, wide enough to span any reasonable possible value of *p*. To show results in a compact format we choose to represent *p* in non-dimensional form as $x = (2p - p_{max} - p_{min})/(p_{max} - p_{min})$. The central value $p_{central}$ equals $(p_{max} + p_{min})/2$, corresponding to x=0. For the large uncertainty range chosen for the parameters 〈, β, γ, ™, and ∑, we find that E_o varies by less than 20% in most cases.

The largest error margins derive from changes in the climate sensitivity, ⊗T_{2X}, and the main parameter used to express climate change damage costs, 〉. Especially for values of *x* close to -1, for these two parameters, E_o could be much larger. For ⊗T_{2X}, however, we find it reasonable not to consider values below 2 °C, given the high likelihood that the climate sensitivity is higher than this level. For 〉, we consider 0.0010 a generous lower bound, as it lies significantly below the minimum of values listed in Table 3 (although Fig.6 extends down to 〉 = 0.0005). Since we abstract from the time-dimension of the cost-benefit problem, we cannot explicitly consider the effect of variations in the discount rate. Indirectly, however, we assume it is included in changes of ρ. Thus, limiting ourselves for ⊗T_{2X} and 〉 down to values of x=-0.5 but up to x=1, corresponding to a broad uncertainty range, we find that the optimal emission level E_o = 8.7 Gt_{CO2}/yr varies by no more than 2.3 Gt_{CO2}/yr or less than 30%.

Fig.6. Dependence of the optimal emissions level E_o (in Gt_{CO2}/yr and as fraction of current emissions E_s) on parameters 〈, ®, γ, ™, ∑, 〉, and ⊗T_{2X} of the optimization problem. Each curve shows the effect of varying the parameter under consideration while keeping the others fixed at their central value. The x-axis shows the variation of each parameter p in non-dimensional form as $x = (2p - p_{max} - p_{min})/(p_{max} - p_{min})$.



5. Cost penalty and value of research

In our cost minimization problem the optimal emission level may well be determined incorrectly as a result of uncertainties in damage and abatement costs. As we have seen, these uncertainties can be large. Consequently, the real cost borne by society when establishing a desirable CO_2 emission level is larger than at the social optimum. Of course, various political processes may also preclude the choice of the optimal emission level, but here we are interested in errors in E_o resulting from erroneously estimated damage and abatement costs. In particular, we examine by how much the total social cost increases above the optimum due to damage and abatement cost estimation errors. Rather than looking at the uncertainty in each of the parameters of Table 5, as we did in Section 4.2, we here take a simplified approach by considering overall errors in respectively the damage and abatement cost.

Suppose that damage costs have been estimated as $C_{dam,est}(E)$, while the true damage cost is $C_{dam,true}(E)$. Likewise, we assume that the abatement cost has been guesstimated as $C_{ab,est}(E)$, whereas it is really $C_{ab,true}(E)$. The optimal emission level corresponding to the estimated costs is $E_{o,est}$, instead of the true optimum $E_{o,true}$. We represent damage and abatement cost uncertainties by random variables, x_{dam} and x_{ab}:

$$x_{dam} = C_{dam,\,true}(E)/C_{dam,\,est}(E), \tag{20}$$

and

$$x_{ab} = C_{ab,\,true}(E)/C_{ab,\,est}(E). \tag{21}$$

We look at variations of x_{dam} and x_{ab} separately because their magnitudes and probability distributions are fundamentally different. Uncertainties in the abatement cost function are much smaller than those in the damage cost. Also, for the former a normal distribution seems most plausible. We therefore characterize the distribution of x_{ab} by a Gaussian with mean 1 and standard deviation σ_{ab}. Since for large σ_{ab} a significant portion of the Gaussian corresponds to negative values of x_{ab}, i.e. negative abatement costs, we truncate the Gaussian at zero and replace it by a normalized distribution that is proportional to the Gaussian at positive x_{ab}.

For uncertainties in the damage cost function we assume a lognormal distribution. A variable has a lognormal distribution if the distribution of the logarithm of the variable is Gaussian. The lognormal distribution is strongly skewed, with a long tail of high values with low probability. It is usually characterized in terms of its geometric mean μ_g and its geometric standard deviation σ_g. Its geometric mean μ_g is equal to the median. If a quantity with a lognormal distribution has a geometric mean μ_g and a geometric standard deviation σ_g, the probability is approximately 68% for the true value to be in the interval $[\mu_g/\sigma_g, \mu_g\, \sigma_g]$ and 95% for it to be in the interval $[\mu_g/\sigma_g^2, \mu_g\, \sigma_g^2]$. Thus the confidence intervals of the lognormal are multiplicative, in contrast to the additive ones of the Gaussian.[10]

[10] See Spadaro and Rabl (2007) for more information on the use of lognormal distributions for the uncertainty analysis of environmental damage costs.

The highly skewed distribution of damage cost estimates in the literature (DEFRA, 2004 and Tol, 2005) is fairly consistent with a lognormal function, even though a few studies claim negative damages. Indeed, global climate change probably produces both winners and losers, at least at moderate temperature increases, but we do not believe that the net world-wide damage cost could be negative for any increase of the atmospheric CO_2 concentration. As representation of the estimates found in the literature we therefore take a lognormal distribution, and choose for its parameters a median μ_g = \$3.8/t_{CO2} and upper limit $\mu_g\ \sigma_g^2$ = \$95/t_{CO2}, that is, $\sigma_g = 5$.[11] In view of the limitations of currently available studies – notably the fact that especially some of the most troubling potential impacts, such as a change in the thermohaline circulation, rapid non-linear ice-sheet disintegration, or methane release from permafrost melting, have not yet adequately or hardly at all been taken into account – we realize that the uncertainty range may well be larger than $\sigma_g = 5$.

Since we focus in this section on variations in x_{dam} and x_{ab}, we use these variables as arguments of the true optimal emission level $E_{o,true}(x_{dam},x_{ab})$ as well as of the difference $\Delta C(x_{dam},x_{ab})$ between the total cost at $E_{o,est}$ and that at $E_{o,true}$:

$$\Delta C(x_{dam}, x_{ab}) = [C_{dam,true}(E_{o,est}) + C_{ab,true}(E_{o,est})] - [C_{dam,true}(E_{o,true}) + C_{ab,true}(E_{o,true})]. \tag{22}$$

$\Delta C(x_{dam},x_{ab})$ is the cost penalty due to errors in the damage and abatement cost functions. The results in this section are complementary to those of Fig.6. Here we cover a wider range of uncertainties than considered there, but present less detail about the specific role of individual parameters. Starting from the quantities $C_{dam,est}(E)$, $C_{ab,est}(E)$, and $E_{o,est}$ as calculated with the central values of the parameters in Table 5, the corresponding true quantities are obtained by incorporating the factors x_{dam} and x_{ab} in our analysis. As can be seen from an inspection of Eqs.8 and 12, the variation of x_{dam} is equivalent to variations in ρ and $(\Delta T_{2X})^2$, while the variation of x_{ab} is equivalent to a variation in α.

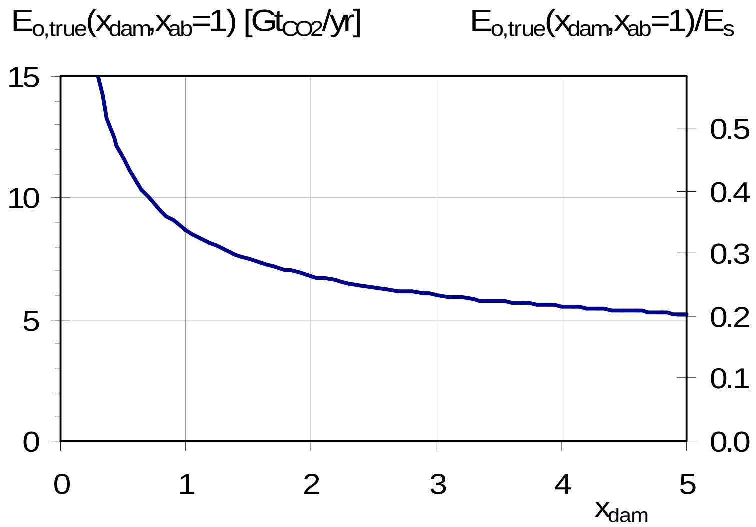
The variation of the true optimal emission level $E_{o,true}(x_{dam},x_{ab})$ is plotted in Fig.7, in part a) as function of x_{dam}, keeping $x_{ab} = 1$, and in part b) as function of x_{ab}, keeping $x_{dam} = 1$. Since we think that the abatement cost uncertainty is smaller than the damage cost uncertainty, the depicted range of x_{ab} is smaller than that of x_{dam} (up to a maximum of 3 and 5, respectively). As expected, both graphs of Fig.7 confirm our central result of E_o = 8.7 Gt_{CO2}/yr, reached in this representation when both $x_{dam} = 1$ and $x_{ab} = 1$. These plots also include our finding that the uncertainty range of this optimal emission level amounts to at most 2.3 Gt_{CO2}/yr, or less than 30%, but furthermore depict the dependence of E_o on a larger scope of damage and abatement cost uncertainties than the ones we considered before, that is, parameter values beyond what we regard generous minimum and maximum boundaries. In addition, through the relative units used for the y-axis on the right, Fig.7 points out that even if the damage or abatement costs are estimated wrongly by as much as a factor of 3, the optimal emission level still amounts to about half the present-day emissions of CO_2. Yet such an error by a factor of 3 could also imply that today's emissions should be reduced by almost 80%, rather than

[11] This median is the one reported in the review by Tol (2005) and the upper limit the 95% confidence interval in this reference.

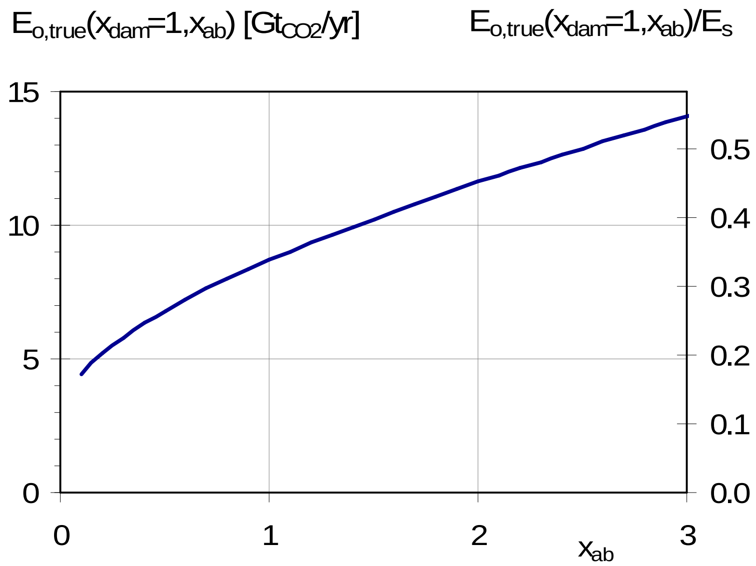
the suggested central reduction value of 67%. These findings strengthen our analysis' case for realizing a deep cut in CO_2 emissions.

Fig.7. Effect of uncertainties on the optimal emission level $E_{o,true}(x_{dam},x_{ab})$.
a) True optimum if true damage cost is x_{dam} times larger than the estimate, keeping $x_{ab} = 1$.
b) True optimum if true abatement cost is x_{ab} times larger than the estimate, keeping $x_{dam} = 1$.

a)



b)



The cost penalty $\Delta C(x_{dam},x_{ab}=1)$ resulting from damage cost errors is shown in Fig.8 as function of x_{dam}, keeping $x_{ab} = 1$. To get a sense for the magnitude of the cost penalty with respect to the total costs incurred at the optimum, it is instructive to complement the cost penalty in absolute terms (solid line and left hand scale) with that in relative terms as ratio $\Delta C/C$ (dashed line and right hand scale), with in this case $\Delta C=\Delta C(x_{dam},x_{ab}=1)$ and $C=C(x_{dam},x_{ab}=1)$. Analogously, the cost penalty $\Delta C(x_{dam}=1,x_{ab})$ due to abatement cost errors is shown in Fig.9, again both in absolute and in relative terms (left and right hand scales, respectively). Like for Fig.7, given that the uncertainty range for abatement costs is probably smaller than for damage costs, we think it justified to depict a smaller x-axis span for x_{ab} than for x_{dam}.

Fig.8. The cost penalty $\Delta C(x_{dam},x_{ab}=1)$ if the true damage cost is a factor x_{dam} times the damage cost estimate, in absolute terms (solid line, left scale) and in relative terms as fraction of the total cost $C(x_{dam},x_{ab}=1)$ at the optimum (dashed line, right scale).

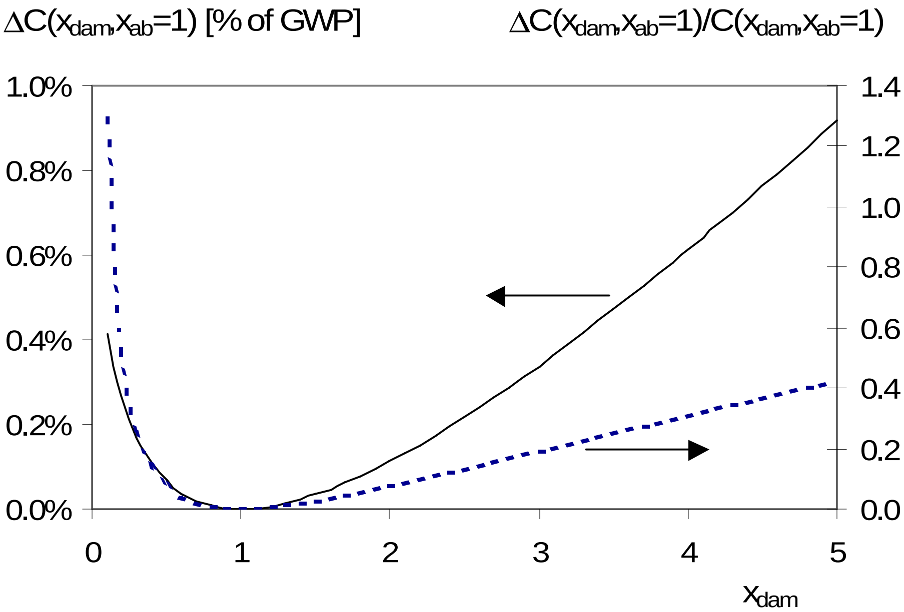
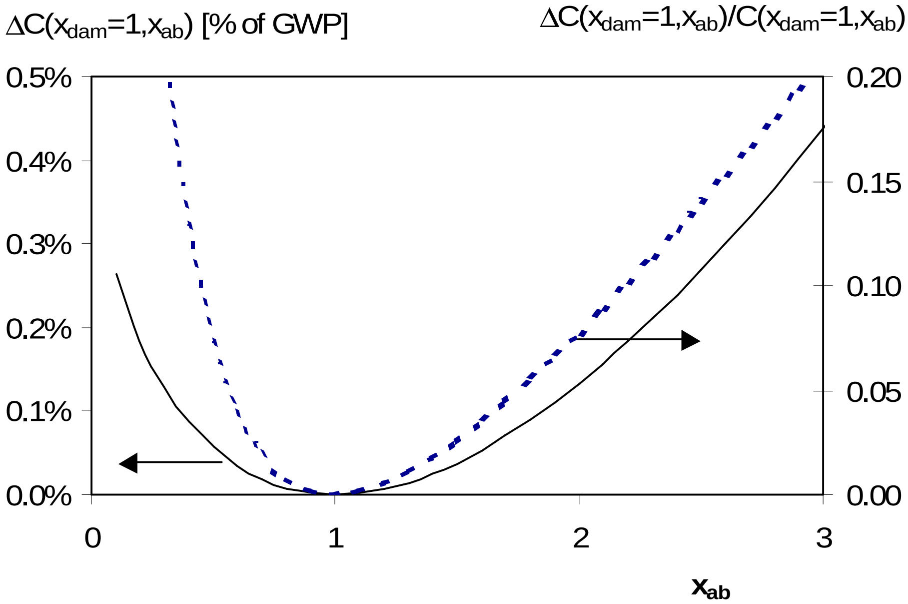


Fig.9. The cost penalty $\Delta C(x_{dam}=1,x_{ab})$ if the true abatement cost is a factor x_{ab} times the abatement cost estimate, in absolute terms (solid line, left scale) and in relative terms as fraction of the total cost $C(x_{dam}=1,x_{ab}=1)$ at the optimum (dashed line, right scale).



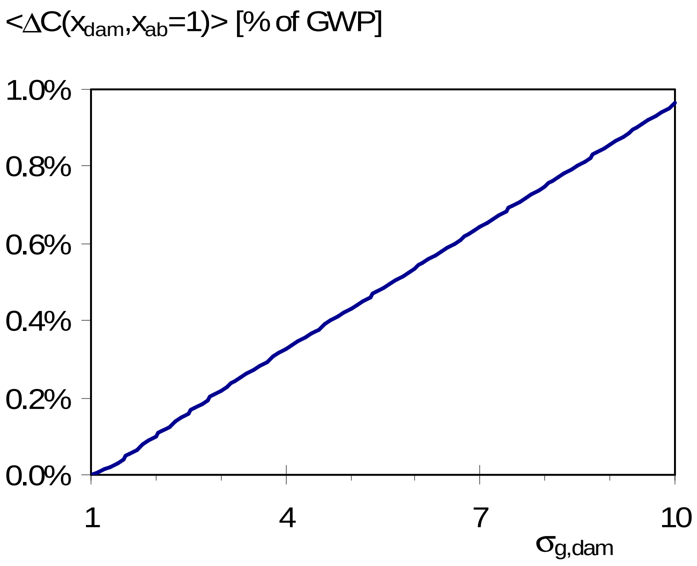
Like in Rabl *et al*. (2005), we find that the cost penalty is relatively small near the optimum, or, in other words, the optimum is fairly broad. Only when the damage and abatement cost errors get large, the cost penalty becomes substantial. Figs. 8 and 9 show that when the true damage costs are 5 times the estimated ones, the relative cost penalty amounts to about 40%, and that when the true abatement costs are 3 times the estimated ones, the relative cost penalty amounts to about 20%. These relative changes may still be considered modest, but in absolute terms the cost penalty can become really enormous. The explanation is that the stakes involved in global climate change, i.e. the costs both at the damage and abatement side of the problem, are simply very high. For example, for x_{dam}=5 and x_{ab} = 3, the cost penalty reaches values nearing 1% and 0.5% of GWP, respectively, close to the total costs of 1.2% of GWP involved in the climate change problem at the optimum (see also Fig.6).

Uncertainties in damage and abatement costs can be reduced by further research. To assess the value of such research we calculate the expectation value $<\Delta C(x_{dam},x_{ab}=1>$ as function of the geometric standard deviation σ_{gdam} of the damage cost, with x_{dam} characterized by a distribution *Lognormal(1,*σ_{gdam}*)*. Likewise, we determine the expectation value $<\Delta C(x_{dam}=1,x_{ab})>$ as function of the standard deviation σ_{ab} of the abatement cost, with x_{ab} characterized by a distribution *Normal(1,*σ_{ab}*)*. Even though abatement options exist with negative costs, we do not believe that on a global scale total abatement costs could be negative. We therefore truncate the normal distribution at x_{ab} = 0. The results are shown in Fig.10.

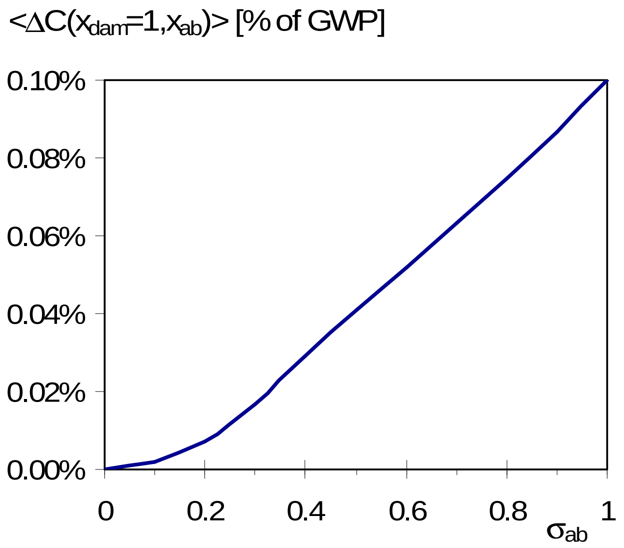
Fig.10 provides an indication of the value of improved information on the damage and abatement costs. For example, if research reduces the geometric standard deviation $\sigma_{g,dam}$ of the relative damage cost estimates from 10 to 5, the expectation value of the cost penalty decreases from around 0.9% to some 0.4% of GWP. Hence the value of such research is about 0.5% of GWP. In relative terms less than half a percent benefit may seem negligible, but in absolute terms this reduction of the cost penalty corresponds to an amount of about 250 billion €. In a similar way, if further research reduces the standard deviation of relative abatement cost estimates from 1 to 0.5, the benefit is about 0.06% of GWP, that is, about 30 billion €. We thus conclude that research to reduce the uncertainty of damage and abatement cost estimates can be extremely cost-effective. We also observe that the possible gains from continued climate change damage cost analyses (i.e. climatic externality studies) may be significantly higher than those obtainable by increasing our understanding of the nature of abatement technologies and their prospected costs. The reason is the small likelihood for extreme climate events with high impacts, that is, the lognormal distribution of the damage cost function. Note that the change of slopes in Fig.10b) around σ_{ab} = 0.2 is a reflection of the truncation at x_{ab} = 0. Such details should not be taken too literally, however, since the probability distributions are not known sufficiently well.

Fig.10. Expectation value of the cost penalty: a) <ΔC(x_{dam}, x_{ab}=1> as function of the geometric standard deviation $\sigma_{g,dam}$ of the damage cost; b) <ΔC(x_{dam}=1, x_{ab})> as function of the standard deviation σ_{ab} of the abatement cost.

a)



b)



6. Conclusions and recommendations

We have carried out a cost-benefit analysis of climate change mitigation, with a focus on the uncertainties associated with both sides of the problem: the damage costs of CO_2 emissions and abatement costs of CO_2 emission reductions. To keep the analysis transparent we have introduced several major simplifications, especially by assuming a time-independent relation between CO_2 emissions and atmospheric CO_2 concentrations, arguing that they do not affect the validity of our conclusions. Based on a review of the literature, we have formulated elementary approximations for the damage and abatement cost functions. For the most plausible choice of the model parameters, we find that the ‘climatic social optimum’ corresponds to an emission level E_o = 8.7 Gt_{CO2}/yr, about a third of CO_2 emissions today.

Varying the model parameters over a wide range, we evaluate the sensitivity of E_o and find that our central result is surprisingly robust. For most of our parameter tests, E_o changes by less than 20%. Varying the climate sensitivity parameter $\otimes T_{2X}$ and the scaling factor ρ of the damage cost function, on the other hand, has a stronger effect. For $\otimes T_{2X}$ we assume a central value of 3°C, a lower bound of 2°C and an upper bound of 5°C. For ρ we adopt a margin broader than the proportionality factors found in the literature, by supposing a central value of 0.0020, a lower limit of 0.0010, and an upper limit of 0.0035. On the basis of the corresponding parameter changes we find that E_o varies by no more than 2.3 Gt_{CO2}/yr, i.e. less than 30% from E_o as found under our central parameter assumptions. This finding both confirms and narrows down the result of the more rudimentary cost-benefit analysis by Rabl *et al*. (2005), who calculate that the optimal CO_2 emission level lies between one third and three quarters of the current emission level under a wide range of parameter choices. Interestingly, our results imply that the optimal emission level is unlikely to be lower than E_o = 6.4 Gt_{CO2}/yr, i.e. about one quarter of current CO_2 emissions, the explanation for which is that the abatement costs become too high at this mitigation plateau.

Ultimately it is not only the optimal emission level and its uncertainty that matters, but also the cost penalty, i.e. the extra social cost incurred due to an erroneously chosen E_o. Wrong choices are likely to occur, given the large uncertainties characterizing the damage and abatement costs that are the main input to our cost-benefit analysis, and the errors they induce in the solution of the problem. Since it proves the optimum is broad, the cost penalty is relatively small even for large errors in the estimation of damage and abatement costs. For example, if the true damage cost is three times larger or smaller than the estimate used in our cost-benefit analysis, the total social cost of climate change increases by less than 20% above its minimum at the true optimal emission level. Because of the magnitude of the total costs involved with global climate change, however, even a fairly small relative error implies a large cost difference in absolute terms, amounting typically to hundreds of billions € in the case of damage cost uncertainties. We have therefore calculated the benefit of reducing these uncertainties. For example, if research reduces the geometric standard deviation of the relative damage cost estimate from 10 to 5, the expectation value of the cost penalty decreases from around 0.9% to some 0.4% of GWP. Clearly, the value of information brought forward by increased climate change damage research can be enormous.

With E_o = 8.7 Gt_{CO2}/yr as optimal central emissions level and an uncertainty range of 2.3 Gt_{CO2}/yr, we derive from Fig.3 an optimal CO_2 concentration of approximately $P_{stab,o}$ = 400 ppmv and a possible variation of some 40 ppmv. This result deviates from the recommendation of Stern *et al*. (2006), who claim that the optimal climate stabilization concentration is around 500 ppmv CO_2 (equivalent) with an error margin of about 50 ppmv. The discrepancy between their and our results is unlikely to be explainable by the fact that Stern *et al*. (2006) aggregate all greenhouse gases, while we only consider the most important contributor to climate change, CO_2. The Stern review has been criticized for several reasons: apart from the low discount rate it employs, notably also for the high damage and low abatement costs it reports. In the present paper we confirm the observation made by others that, despite the Stern review’s high damage and low abatement costs, it arrives at an inexplicably high climate stabilization level. We agree with the Stern review's overall conclusion that a deep cut in CO_2 emissions is required to avert the risk of global climate change. Like others, however, we question certain aspects of the analysis leading to this result. In particular, we find in our analysis that the marginal damage costs (77 €/tCO_2 at E_s and 54 €/tCO_2 at E_o) are well below those quoted in the Stern review (85 €/tCO_2). Our suggested CO_2 concentration level is not only significantly lower than that of Stern *et al*. (2006) but also than recently professed policy statements, targeting typically at values around 500 ppmv.

From the above climate change cost-benefit analysis, and the description of the uncertainties involved, it is evident that much more work is required in the field of CO_2 damage and abatement cost calculations. Especially climate change damage research really has only barely started off. In order to reduce damage cost uncertainties and exploit the value of corresponding information, it is particularly important to perform detailed analyses of regional climatic impacts and associated economic costs. These are urgently needed to complement the highly aggregated studies produced so far, like the one presented in this paper. We thus agree with the recommendation by DEFRA (2004) that the disaggregation and valuation of damage costs by sector and region should be forcefully pursued. Determining the possible physical impacts of CO_2 emissions in all areas of economic and social activity should be vigorously continued, since the ensuing findings can effectively profit long-term policy making. The classical challenges of mitigation timing, social discounting, equity weighting, and risk aversion remain on the agenda, as well as the question how policy makers should confront the uncertainties associated with climate change damage and CO_2 abatement costs. To the latter, this article has attempted to contribute a step forward. As in the future more understanding on all the above fields emerges, the type of analysis presented here should be revisited.

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