Why Intergenerational Externalities Matter for Sustainable Growth*

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Abstract

A common view is that economic growth and high environmental quality can coexist with ongoing technological progress. This view, however, neglects the intergenerational externalities that arise when individuals ignore the effect of their actions on future generations. I examine how intergenerational externalities distort the demand for clean technologies and why this might be critical for sustained economic growth. I show how a strong preference for one's economic growth can be harmful to future economic growth. Furthermore, I show why the demand for clean technologies can be insufficiently low to ensure sustained economic growth regardless of the strength of individual's preferences for environmental quality.

Keywords: intergenerational externalities, demand for clean technologies, implementation of clean technologies, sustainability, economic growth

JEL Classification Codes: O44, Q01, Q5

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1 Introduction

In recent work, the United Nations Environment Program (UNEP) highlighted the role clean technologies must play if we are to enjoy both economic growth and reduced pollution levels (UNEP, 2009). While investment in clean technologies has increased considerably since 2000,[1] their implementation lags. For example, energy from commercially available renewables only accounts for 19% of global energy consumption.[2] This relatively small share is surprising because there are many technically proven clean technologies available but they are as yet under utilized.[3] In this paper, I examine how intergenerational externalities may lower the demand for clean technologies producing a similar situation.

A common view is that to ensure the well being of future generations, the current generation must trade-off economic growth and environmental quality. While this may be true, the costs of sustainable growth are lower when ongoing technological progress provides an innovative set of clean technologies. Unfortunately, the majority of both the classical growth and the new endogenous-growth literature focus on the supply of these new technologies taking as given society's willingness to pay for their implementation. But availability, in general, does not guarantee their use. In contrast, I focus on the demand side and, in particular, how intergenerational externalities distort the demand for clean technologies.

It is well known that current and future generations have limited opportunities for trade or coordination of policies. In the context of sustainable growth, these limitations may have stark consequences since they lower the incentive to innovate clean, long lasting, technologies whose primary benefit falls on future generations. In particular, I consider self-interested individuals who neglect the effect of their actions on later generations and only demand clean technologies for their benefit. In doing so, I show why this might be critical and why supply-side considerations alone may not ensure sustained economic growth.

I develop my results in an endogenous growth model with overlapping generations (OLG). Consumers invest in capital and demand clean technologies, while they get utility from consumption and environmental quality. On the production side, agents engage in research and the resulting innovations improve the quality of existing goods through creative destruction. Aggregate output is a function of capital, labor and a technology index. Although output generates pollution, consumers can offset this effect by employing cleaner technologies.

Within this context, I examine how intergenerational externalities can lead to several surprising results. First, I show that no-growth and environmental degradation are a possible equilibrium outcome. One may expect that economic growth is fast in economies where agents care little about the environment. However, I show that when agents care little about the environment, environmental quality not only deteriorates but economic growth can be negative. This result deviates the common view drawn from related work where in the presence of endogenous technological progress both economic growth and environmental quality improve (e.g., (Bovenberg & Smulders, 1995), (Aghion & Howitt, 1998a)).

Second, I show that in a situation with even moderate natural regeneration, a stronger preference for a cleaner environment will lead to both faster growth and an improved environment. Innovation and capital accumulation generate income for consumption but since the possibilities to innovate are constrained and there are diminishing returns to capital, consumers cannot only consume but must also demand clean technologies. As each generation is born richer, with both a higher stock of knowledge and capital, they allocate a higher share of their income to environmental quality. Hence, faster innovation and capital accumulation are associated with cleaner technologies. This implies a complementarity between economic growth and environmental quality. In contrast, it is typical to find growth and environmental quality to be substitutes in the literature (e.g., Aghion & Howitt (1998a)).

Third, I find that environmental degradation may occur regardless of the strength of individuals' preference over environmental quality. This differs from the conventional wisdom that environmental problems arise when individuals care too little about the environment (e.g., Bovenberg & Smulders (1995)). But if individuals only care about environmental quality while they are alive, a stronger preference for environmental quality can only increase the demand for clean technologies to benefit their own generation but they ignore the negative effect of pollution on the future. Therefore, environmental problems, in my context, do not arise because individuals care too little about the environment, but because they care too little about their children.

Fourth, I find that the distortion in the demand for clean technologies leads to environmental degradation. This deviates from the common view drawn from the endogenous growth literature that sustained growth is possible with technological progress in clean technologies (e.g., Smulders & Gradus (1996), Acemoglu et al. (2009)). As consumers do not care about the effect of capital accumulation on future pollution, their demand for clean technologies is not sufficiently high to offset the negative effect on the environment.

There is an extensive literature examining the relationship between economic growth and environmental quality.[4] My paper closely relates to two main strands of the literature: the literature examining sustainable growth in a world with endogenous economic growth, and the literature looking at intergenerational externalities in environmental quality. While these literatures are large, existing work addresses each issue separately. I contribute to the literature by examining them jointly.

The endogenous growth literature emphasizes how the supply of innovations and directed technical change can reconcile economic growth within environmental constraints (e.g., Aghion & Howitt (1998a), Gradus & Smulders (1993), Bovenberg & Smulders (1996), Smulders & Gradus (1996), Acemoglu et al. (2009)). By relaxing this literatures standard assumption of infinitely-lived agents, I find that an economy's transition from dirty to clean technologies, that offsets the negative effect of pollution, might not occur because of the insufficient demand for clean technologies. Hence, I focus on another difficulty we face in achieving sustainable growth.

The second branch of the literature analyzes how intergenerational externalities affect environmental outcomes but without explicitly considering sustainable growth. While (John & Pecchenino, 1994) and (John et al., 1995) explore how distortions in abatement decisions affect environmental quality, (Von Amsberg, 1995) and (Howarth, 1991) show how the incompleteness of insurance markets can lead to poor environmental outcomes. And in the resource context, (Howarth & Norgaard, 1990, 1992) explore how the efficient resource extraction path is affected by the distribution of rights and assets across generations. My focus on how intergenerational externalities affect the transition from dirty to clean technologies and therefore long run growth, distinguishes my paper from related work.

The paper is organized as follows. Section 2 presents a Schumpeterian model with environment in an overlapping generations framework and section 3 analyzes the equilibrium results. In section 4, I analyze the problem of a social planner. Section 5 concludes.

2 An endogenous growth model with OLG

The core analytical framework is the Schumpeterian model of (Aghion & Howitt, 1998a), except that I consider an overlapping generations (OLG) demographic structure. The OLG structure allows me to model selfish individuals who ignore how their actions affect future generations. I focus on how the lack of coordination between generations on their demand for consumption and environmental quality challenges the possibilities for sustained economic growth. By only relaxing the infinitely lived agents' assumption, I pin down why current generations might implement clean technologies slowly and how it reduces the chances for sustained economic growth. While the incentives to free ride on the provision of environmental quality for the future might be known, I am not aware of previous papers that point out the challenges for the environment and economic growth simultaneously.

There are two economic agents: producers and consumers. I start out by describing the production process and next, I examine the demand for consumption goods and environmental quality of finitely lived consumers.

2.1 Producers

I develop a multiproduct Schumpeterian growth model with capital where individuals have a finite lifetime. I illustrate the sketch of the model in figure 1. There are three sectors: research and development (R&D), intermediate goods and final goods. The research sector provides new innovations to the intermediate sector, which produces goods for the final sector.

The only difference between the OLG model I develop here and the usual endogenous growth models with infinitely lived agents is the lifetime of patents. Those who enter the R&D sector engage in uncertain research.[5] If research is successful, the newest innovator earns the right to start up a monopolistic firm that exclusively produces a given intermediate good. Since I assume that patents last for one generation, the newest innovator earns monopoly profits for one period. After that period, competitive firms produce the intermediate good.[6] This assumption is consistent with current US and European patent laws that limit the exclusive rights of new innovations.[7] If research is unsuccessful, competitive firms produce intermediate goods with existing technology.

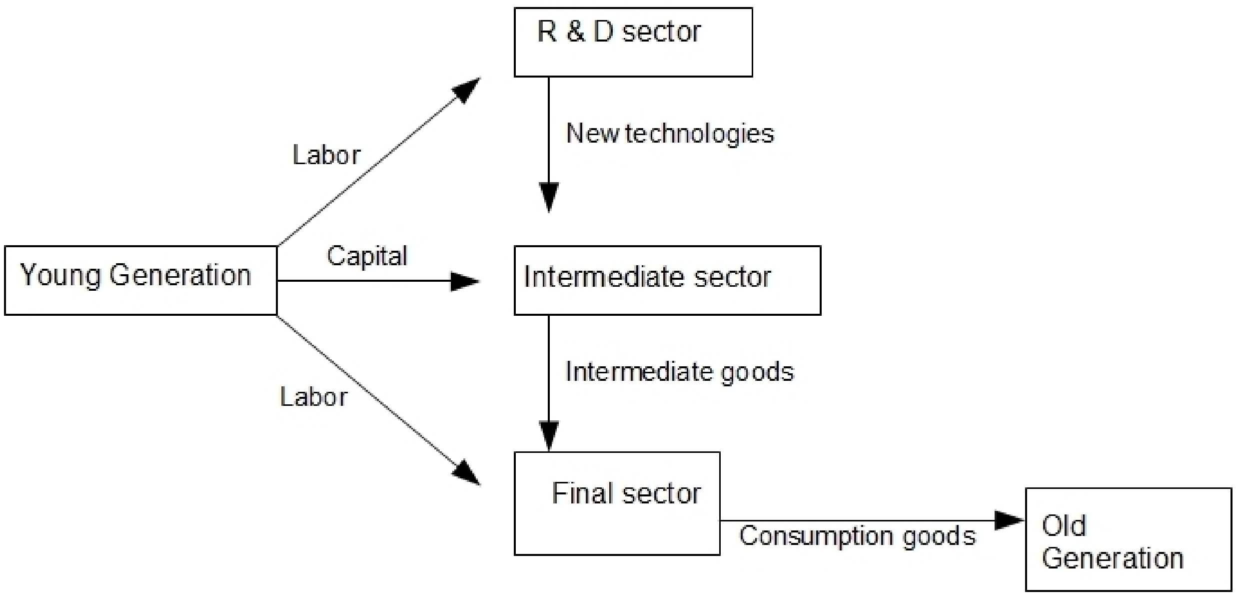

Figure 1: Production process in a Schumpeterian growth model with finite lifetime of individuals

The rest of the production process I describe below is as in (Aghion & Howitt, 1998a).

2.1.1 R&D sector

There is a different research sector for each intermediate good where several firms compete for new discoveries. Since the newest successful innovator earns the right to exclusively produce the intermediate good, each competitive firm maximizes the expected profits of starting up a monopolistic intermediate firm: $\max_{L_{R,it}} \quad \mu_i \pi_{i,t} - w^R_{i,t} L_{R,it}$, where μ_i is the number of successful innovations, $\pi_{i,t}$ is profits in the intermediate sector, and $w^R_{i,t}$ and $L_{R,it}$ are wages and workers. The likelihood of new innovations improves with the number of researchers and their productivity $\lambda > 0$; i.e., $\mu_i = \lambda L_{R,it}$.[8]

The total number of innovations is the sum of all researchers and their productivity so that $\mu = \int_0^\infty \lambda L_{R,it} di = \lambda L_{R,t}$.[9] This is so because the same number of people work in each sector. Since all innovations draw on the same pool of shared knowledge, the prospective payoffs to research is the same in all sectors. The general knowledge improves with each new discovery, even though innovations are independent and only implementable in the innovator's sector. Thus, the latest innovator discovers a marginally better technology than the existing ones and improves the leading edge technology $A_t^{\max}$ by the constant factor $\varphi > 1$.[10]

The R&D sector provides newer and more productive innovations to the intermediate good sector. Technological progress takes the form of creative destruction, where quality-improving innovations make existing innovations obsolete. While the new innovators become the exclusive monopolistic producers of intermediate goods in their own sectors, the sectors with unsuccessful innovators have competitive firms producing intermediate goods.

2.1.2 Intermediate sector

Intermediate firms supply multiple intermediate goods to the final sector to be used independently as inputs. The only input in the production of intermediate goods is capital and there are two types of intermediate firms: monopolistic and competitive. As shown above, while monopolistic firms are the latest innovators from R&D, competitive firms belong to sectors with unsuccessful innovations. Since the number of innovators is μ, there are μ monopolistic and $1-\mu$ competitive firms.

We can think of the monopolistic producer as an innovator who sets up a new firm following his innovation. The incumbent monopolist produces the intermediate good $x_{i,t}^m$ to maximize profits $\pi_{i,t}$: $\max_{x_{i,t}^m} \quad \pi_{i,t} = p_{i,t}^m(x_{i,t}^m)x_{i,t}^m - r_t x_{i,t}^m A_{i,t}^{\max}$, where r_t is the rental price of capital. The price of the good is the inverse demand curve facing the latest innovator, $p_{i,t}^m(x_{i,t}^m) = \frac{\partial Y_i}{\partial x_i^m}$. Since all successful innovations happen at the same time and use the leading edge technology, their price is $p^m = \frac{rA^{\max}}{\alpha}$.[11]

Competitive firms are all sectors with unsuccessful innovations and sectors with expired patents. They produce the intermediate good $x_{i,t}^c$ to maximize profits: $\max_{x_{i,t}^c} \quad \pi_{i,t} = p_{i,t}^c x_{i,t}^c - r_t x_{i,t}^c A_{i,t}$, where $p_{i,t}^c$ is the competitive price for the final sector $p_{i,t}^c = rA_i$. The price of each intermediate good vaires with the productivity of technology.

Capital is exclusively used in the intermediate sector. This implies a capital stock equal to the productivity of intermediate goods: $K_t = \int_0^1 A_i x_i di$. Since there are μ monopolistic and $1-\mu$ competitive firms, the stock of capital is $K_t = \mu_t A_t^m x_t^m + (1-\mu_t)x_{i,t}^c \int_0^1 A_{i,t} di$.

2.1.3 Final sector

Perfectly competitive firms hire labor and a continuum of intermediate goods to produce the final output: $\max_{x_{i,t},L_{Y,t}} Y_t - \int_0^1 p_{i,t}x_{i,t}d_i - w_t^Y L_{Y,t}$, where the price of the intermediate good i, $p_{i,t}$, and wages, w_t^Y, are takes as given. Y_t is the aggregate output with production process:

$$Y_t = L_{Y,t}^{1-\alpha} z_t \int_0^1 A_{i,t} x_{i,t}^{\alpha} d_i, \tag{1}$$

where $L_{Y,t}$ is workers in the manufacturing sector, z_t a clean technology index, $A_{i,t}$ is a productivity parameter of the latest version of intermediate product i, and $x_{i,t}$ is intermediate good i. The key feature of this production function is the clean technology index z_t I present below.

2.1.4 The production process and the environment

Below I explain how the environment affects the production process. There are three features to highlight. First, environmental quality is not an input in the production process, which implies that environmental quality only affects consumers' utility. Second, there is no directed technological change, which means that all innovations improve the productivity of the economy. Thus there is no distinction between research effort directed to clean and dirty technologies. Third, a clean technology index z_t, first introduced by (Copeland & Taylor, 1994), affects the productivity of the production process.

We can think of this technology index as pollution intensity, an index of the emissions rate for the production process (Copeland & Taylor, 1994).[12] This index is between 0 and 1. When the index is close to 1, the technology is dirty and productive, and when it is close to zero, the technology is clean but less productive. Clean technologies are substitutes of intermediate goods and labor in the production of final goods. A production process with cleaner technologies uses more intermediate goods and labor input. This set up implies that while there are clean and dirty technologies available in the economy, the implementation of clean technologies is given by consumer demand. Firms employ the amount of clean technologies that individuals demand. I assume that

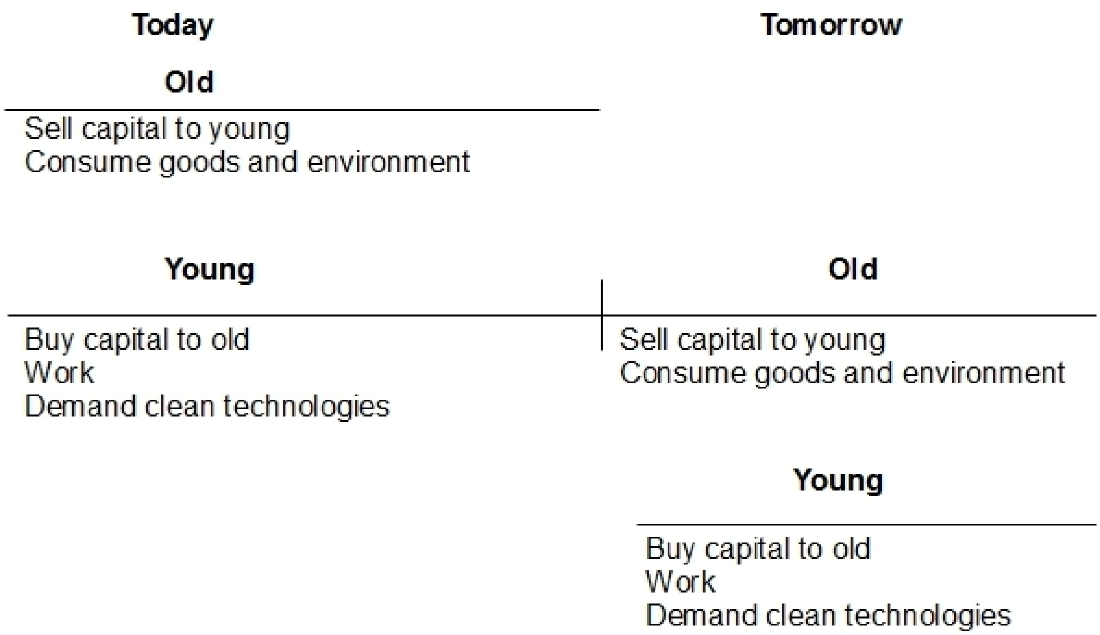

Figure 2: The problem of consumers and generational links

within each generation there are no free-rider issues and the demand for clean technologies coincides with the socially desirable level. I describe the demand for clean technologies in more detail below.

2.2 Consumers

Consumers get utility from consuming goods and from environmental quality. While the production process generates pollution, consumers can offset this effect by demanding clean technologies. My focus is to study the implications of the demand for clean technologies on sustainable growth and I do this by contrasting to others in the literature, who focus on the supply of clean technologies to guarantee sustainable growth.

2.2.1 Utility and budget constraints

Finitely lived agents care about their own well-being but neglect the effects of their actions on future generations. I illustrate the model in figure 2. A new generation is born at the beginning of each period and lives for two periods.[13] Consumers' get utility from consumption and environmental quality:

$$U = U(c_{o,t+1}) + \phi V(E_{t+1}), \tag{2}$$

where $U(.)$ and $V(.)$ are the utility from consumption and from environmental quality, $c_{o,t+1}$ is consumption when old, E_{t+1} is environmental quality when old, and ϕ is the weight consumers place on environmental quality relative to consumption. For simplicity, the utility is logarithmic, ($U(.) = V(.) = \ln(.)$). Note that consumers only enjoy consumption and environmental quality when old. This simplifying assumption is standard in the environmental economics literature to abstract from *intra*generational free-rider problems and to focus on *inter*generational free rider issues (e.g., John & Pecchenino (1994)). This implies that since only one generation consumes in each period, there is no strategic interaction between different generations. Hence equilibrium results pin down the effects of intergenerational externalities on sustainable growth.[14]

All young people work in exchange of a wage. Since they only consume when old, they save all their earnings when young. These savings are used to purchase capital from the old, who are ready to retire and employ these returns to pay for consumption.[15] Thus, the budget constraints of the young and old are:

$$w_t = s_t \tag{3}$$

$$c_{o,t+1} = (1 + r_{t+1})s_t \tag{4}$$

where w_t, s_t and r_t are wages, savings and interest rate. Since individuals are not altruistic, they end up with no assets at death.[16] Hence, all savings equal next period's capital stock so that $K_{t+1} = s_t$. This implies that the savings and capital investment decisions are equivalent in equilibrium.[17]

2.2.2 The environment

The stock of environmental quality improves naturally at rate b and degrades with the flow of pollution $P(Y, z)$:

$$E_{t+1} = (1 + b)E_t - P(Y_t, z_t), \tag{5}$$

where $0 < E_t < E_{\max}$ and $b > 0$.[18] Pollution is a byproduct of production, $P(Y_t, z_t) = Y_t z_t^{\gamma}$, where Y_t is aggregate output in (1) and γ the weight of clean technologies in pollution. In absence of pollution, the stock of the environmental quality reaches a maximum $E_{\max}$.[19]

2.2.3 Consumers' problem

Young individuals maximize their lifetime utility (2) demanding clean technologies z_t and choosing their saving s_t while taking as given the budget constraints, (3)-(4), environmental quality (5), wages w_t, and the interest rate r_t. The consumers' maximization problem then becomes:

$$\begin{aligned} \max_{s_t, z_t} \quad & U(c_{o,t+1}) + \phi V(E_{t+1}) \\ \text{s.t.} \quad & w_t = s_t \\ & c_{o,t+1} = (1 + r_{t+1}) s_t \\ & E_{t+1} = (1 + b) E_t - Y_t z_t^{\gamma}. \end{aligned}$$

Note that consumers can demand clean technologies to offset the negative effect of capital accumulation on pollution. The demand for clean technologies captures the trade-off between consumption and environmental quality. Dirtier technologies are more productive, generating higher wages for young people but reducing their utility due to lower environmental quality when they become old. Cleaner technologies are associated with higher utility from environmental quality but lower income. While there is empirical evidence on consumers' willingness to pay for green products (Kahn (2007), Schlegelmilch et al. (1996), and Thompson (1998)) and renewable energy (e.g., Roe et al. (2001), Zarnikau (2003)), it is not clear that the demand is large enough to maintain sustained economic growth. In this paper I show that a reason why this might be the case is that intergenerational externalities lower the demand for clean technologies delaying so the implementation of readily available clean technologies.

2.2.4 Intergenerational externalities and the demand for clean technologies

In absence of regulation, there are *intra*-generational and *inter*-generational externalities. Members of each generation free ride on the use of clean technologies when they ignore the benefits on others (*intra*generational externality) and future generations (*inter*generational externality). Since my focus is on long run sustained economic growth, I analyze *inter*generational externality and I abstract from *intra*generational externalities. To do so, I consider a myopic government that internalizes the *intra*generational externality while ignoring distortions on future generations. This assumption is in line with the public policy literature where it is common to assume a government that internalizes an externality while ignoring other distortions (e.g., John et al. (1995), Jones & Manuelli (2001)).[20]

Since there is a government that correctly internalizes the intragenerational externality, each individual demands the socially desirable level of clean technologies for their generation. The capital investment decision is however an individual decision. Thus, individuals neglect the effect of their demand for clean technologies on future generations. This feature differentiates this paper from closely related papers that consider infinitely lived agents. The decision makers in Stokey (1998) and Aghion & Howitt (1998a) are social planners and, hence, they consider the implications of clean technologies and capital accumulation both on current and future generations. [21]

3 Market Equilibrium

The key aspects of the model presented in the previous section are the following. Firms' choose the speed of innovations, while consumers' choose capital accumulation and the implementation of readily available clean technologies. Also, environmental quality deteriorates with pollution. This boils down to explaining this economy with the dynamic behavior of four variables: innovation, capital accumulation, clean technologies, and environmental quality. I present all calculations in the appendix. Let us start with the definition of the equilibrium.

An equilibrium in this economy consists of sequences of the number of workers in each sector, their wages, flow of innovations, productivity of technology, capital, price of capital and intermediate goods, clean technologies and consumption ($L_{R,t}, L_{Y,t}$, w_t^R, w_t^Y, μ_t, A_t, K_t, r_t, p_t, z_t, C_t), such that, in each period: i) firms maximize profits choosing the number of workers and capital; ii) consumer's maximize their utility choosing savings and clean technologies; iii) wages and prices clear the labor, capital and intermediate good markets ($w_{Y,t}, w_{R,t}$, p_t, r_t); iv) consumers' savings equal the capital stock, $s_t = K_{t+1}$; v) the goods market equilibrium, $Y_t = C_t + K_{t+1} - K_t$, is satisfied.

3.1 The Balanced Growth Path

As mentioned, innovation, capital accumulation, clean technologies, and environmental quality describe the economic system. Let us start with innovations. The labor market clearing conditions set the number of innovations every since the number of workers are the only input in the production of new innovations. Every new innovation improves the overall productivity of the economy, and thus we can understand innovation driven economic growth studying the labor market clearing.

Firms choose the number of workers that maximize profits both in the research and manufacturing sectors (eq. 2.1.1 and 2.1.3). The number of workers in each sector that clear the labor market are:

$$L_R^* = \frac{\lambda \alpha^{\frac{1}{1-\alpha}} a - 1}{\lambda((1+\frac{1}{\alpha})\alpha^{\frac{1}{1-\alpha}} a - 1)}, \quad (6)$$

$$L_y^* = 1 - L_R^*. \quad (7)$$

where λ is the productivity per research worker, $1-\alpha$ is the weight of workers on the production of final goods and $a = \frac{A^{\max}}{A}$ is the productivity improvement of the newest innovations. As described earlier, the equilibrium number of workers in research (eq. 6) establish the new set of innovations. These new set of innovations improve the leading edge technology:

$$A_{t+1}^{*\max} = (1 + \mu_t^* \varphi) A_t^{\max}, \tag{8}$$

where $A_t^{*\max}$ are the leading edge innovations, $\mu_t^* = \lambda L_{R,t}^*$ is the flow of new innovations and $\varphi > 0$ is the factor of proportionality. Let us differentiate between φ and a. While φ is the productivity improvement of new innovations from one generation to the next, a is the productivity improvement within a generation. The growth rate of the leading edge innovations is proportional to the aggregate flow of innovations (μ_t^*) with factor of proportionality equal to φ. We can also think of (8) as the law of motion governing the evolution of social knowledge (Aghion & Howitt, 1992). Since the number of workers in research is constant over time (6), the probability of an innovation is also constant.[22] The probability of innovations must be $\mu^* = \lambda L_{R,t}^* \in (0, 1)$ and to ensure this, I assume $\lambda < 1 + \frac{1}{\alpha}$.

The quality improvement of the average innovation drives the total productivity of the economy. In particular, the evolution of productivity depends on the number of new innovations and their quality improvement, $A_{t+1} = A_t \left(1 + \mu(a_t - 1)\right)$. Substituting the equilibrium number of innovations, μ^*, the evolution of aggregate productivity is:

$$A_{t+1} = A_t \left(1 + \frac{\lambda \alpha^{\frac{1}{1-\alpha}} a_t - 1}{(1 + \frac{1}{\alpha}) \alpha^{\frac{1}{1-\alpha}} a_t - 1} (a_t - 1) \right). \tag{9}$$

After I describe the innovation process in equilibrium, I turn to capital accumulation and clean technologies. These two are key elements relate to consumers' choices. Recall that the focus of my work and what distinguishes these results from others in the literature is on the study of the effects of consumers' intergenerational externalities on long run growth. The impact of intergenerational externalities on capital accumulation and the demand for clean technologies are therefore key here.

Young individuals save all their income for future consumption (eq. 3). Also, recall from the previous section that consumers' assets equal the capital stock, $s_t = K_{t+1}$. Since consumers save all their wages when young, the marginal propensity to save characterizes capital accumulation in the economy:

$$K_{t+1} = (1-\alpha)\frac{Y_t}{L_{Y,t}}. \tag{10}$$

A key element to note is the lack of bequests from one generation to the other. I relax this assumption when I consider altruistic individuals in the last section. Note also that in this model households consume all their assets before they die. This implies that consumers are not allowed to discard some of their assets when they are old and before they die.

Let us next look at the implementation of clean technologies. The demand for clean technologies captures the individual trade-off between consumption and environmental quality:

$$U_1'(c_{o,t+1})\frac{\partial c_{o,t+1}}{\partial z_t} - U_2'(E_{t+1})\frac{\partial E_{t+1}}{\partial P_t}\frac{\partial P_t}{\partial z_t} \quad = \quad 0. \tag{11}$$

Since households only consume when old, this demand for clean technologies is an intratemporal decision between consuming goods and enjoying environmental quality. This lack of strategic interactions allows us to pin down the effect of intergenerational externalities on long-run growth. While cleaner technologies generate lower wages due to lower productivity, they improve environmental quality through lower pollution. The social demand for clean technologies ensures the balance between private consumption and environmental quality:

$$z_t^{\gamma} = \frac{(1+b)}{1+\phi(1+\gamma)}\frac{E_t}{Y_t}, \tag{12}$$

where E_t and Y_t are environmental quality and aggregate output. The marginal rate of substitution, which describes the improvement in environmental quality needed to compensate for lower consumption, and the marginal rate of transformation, which captures the reduction in income that improve environmental quality, characterize the demand for clean technologies in equation (12).

The focus of this paper is on analyzing the long-run balanced growth path (BGP) along which all variables grow at constant rates. To do so, I rewrite innovation (9), capital accumulation, the clean technologies (12) and environmental quality (5) in growth rates. I use these four equations to form a system of four equations on four unknowns: innovation, capital, clean technologies and environmental quality (g_A, g_K, g_z, g_E).

First, the growth rate of innovation is:

$$1 + g_A = 1 + \varphi \left(\frac{\lambda \tilde{\alpha}(1+\varphi) - 1}{\lambda((1+\frac{1}{\alpha})\tilde{\alpha}(1+\varphi) - 1)} \right), \tag{13}$$

where $\tilde{\alpha} = \alpha^{\frac{1}{1-\alpha}}$. As described earlier, the number of workers allocated to the research sector establishes the speed of innovation. As the number of workers allocated to each sector is constant over time, the growth rate of innovation is also constant.

Second, from aggregate production in 1, I derive the relationship between clean technologies, innovation and output growth:

$$1 + g_z = \left(\frac{1+g_K}{1+g_A} \right)^{1-\alpha}. \tag{14}$$

Faster innovation generates a higher use of clean technologies. This is consistent with the literature on economic growth and the environment (e.g., Smulders & Gradus (1996), Acemoglu et al. (2009)) and the empirical evidence on environmental kuznets curves.

Third, I derive trade-off in growth rates between consuming goods and environmental quality from the consumers' intratemporal trade-off in (12):

$$(1+g_K)^{\frac{1+(1-\alpha)\gamma}{1+\gamma}} = (1+g_E)^{\frac{1}{1+\gamma}} (1+g_A)^{\frac{(1-\alpha)\gamma}{1+\gamma}}. \tag{15}$$

This implies a complementarity between economic growth and environmental quality for consumers. Innovation and capital accumulation generate income for consumption but since the possibilities to innovate are constrained and there are diminishing returns to capital, consumers cannot only consume but must also demand clean technologies. As each generation is born richer, with both a higher stock of knowledge and capital, they allocate a higher share of their income to environmental quality. Hence, faster innovation and capital accumulation are associated with cleaner technologies.

Fourth, from the evolution of environmental quality in (5), I derive the relationship between economic growth, environmental quality and the demand for clean technologies in

$$1 + g_E = (1 + g_Y)(1 + g_z)^{\gamma}. \tag{16}$$

This implies a complementarity between the growth rate of output and environmental quality. The complementarity is consistent with the data. Grossman & Krueger (1995) find no empirical evidence that environmental quality deteriorates steadily with economic growth. Rather, they find that for most indicators, economic growth brings an initial phase of deterioration followed by a subsequent phase of improvement.

The balanced growth path (BGP) is the solution to the system of four equations in four unknowns that solves for the growth rates of innovation, capital, clean technologies and environmental quality (g_A, g_K, g_z, g_E). I interpret the results in the next section.

3.2 Results: Sustainable Growth in the BGP

The BGP market equilibrium is the solution to the system of equations (13, 14, 15, 16) describing the relationship between the growth rates of innovation, the environment, clean technologies and aggregate output. My focus is on studying the possibilities for sustainable growth in the BGP and on learning how today's consumers might reduce the chances of a sustained economic growth for the future. Let us start by defining sustainable growth. I follow the UNEP definition of sustainable growth: "improving the quality of human life while living within the carrying capacity of supporting ecosystems" (Munro et al. (1991)).[23] Formally:

Definition 1. Sustainable growth is improving economic growth without deteriorating environmental quality $(g_Y \geq 0, g_E \geq 0)$. Unsustainable growth can take two forms. One, improving economic growth with deteriorating environmental quality $(g_Y \geq 0, g_E \leq 0)$. And two, deteriorating economic growth and deteriorating environmental quality $(g_Y \leq 0, g_E \leq 0)$.

Let us start by analyzing the equilibrium growth rate of innovation in (13). The share of the population working in the research sector is constant over time (6). As the number of workers determine the number of innovations, the speed of innovation is time invariant and independent of the growth rate of aggregate output and environmental quality. Therefore, technological progress improves at a constant rate over time (13). While both the productivity per worker and the quality improvement in new technologies increase the growth rate, the weight of capital in production reduces the speed of growth.

Second, the growth rate of environmental quality is also time invariant in equilibrium. There are two possible equilibrium solutions. First, the growth rate of environmental quality (16) and the social demand for clean technologies (12) lead to:

$$1 + g_E = \frac{\phi(1+\gamma)(1+b)}{1+\phi(1+\gamma)} \tag{17}$$

where the growth rate of the environment depends on the preference for environmental quality (ϕ), environmental regeneration rate (b) and the weight of clean technologies on pollution (γ). As described earlier, since every generation only consumes once in their lifetime, the demand for clean technologies is an intratemporal decision for individuals. Since every consequent generation behaves the same way, the growth rate of environmental quality is independent of the evolution of income overtime.

Finally, I derive the relationship between economic growth and environmental quality. Technological progress and capital accumulation generate economic growth constrained by environmental quality. I derive an equilibrium relationship between technological improvement, clean technologies and environmental quality combining equations (13)-(16):

$$1 + g_E = (1+g_K)^{\gamma(1-\alpha)-1}\,(1+g_A)^{\gamma(\alpha-1)} \tag{18}$$

which implies a complementarity between economic growth and environmental quality in the BGP equilibrium.

As discussed in section 2.2, environmental quality has both a lower and an upper bound; $E_t \in (0, E^{\max})$. Therefore, when environmental quality improves in the BGP, it reaches the upper bound $E = E^{\max}$. And when environmental quality declines in the BGP, it reaches the lower bound $E = 0$. Environmental quality will improve over time if and only if $b\phi > \frac{1}{1+\gamma}$. This leads to the main results of the paper summarized in proposition 1 and 2.

Proposition 1. *For any feasible parameter values $(\alpha, \lambda, \gamma, \varphi, b)$, there exists a preference for environment $\bar{\phi} > 0$ such that, for all $\phi > \bar{\phi}$, the BGP equilibrium exhibits sustainable economic growth. In the BGP equilibrium environmental quality improves and it reaches the maximum ($E = E^{\max}$, $g_E^* = 0$). Furthermore, there is positive economic growth ($g_Y^* > 0$).*

Let us first analyze the BGP equilibrium when environmental quality improves (proposition 1). I show that in a situation with even moderate natural regeneration, a stronger preference for a cleaner environment will lead to both faster growth and an improved environment. In this case, economic growth is sustainable in the BGP equilibrium. Since there is an upper bound in environmental quality, the environment reaches the upper bound ($E = E^{\max}$). This implies that in the BGP equilibrium, the growth rate of environmental quality is zero. Hence, aggregate output grows a the constant growth rate $(1 + g_Y) = (1 + g_A)^{\gamma(\alpha-1)} 1 - \gamma(1 - \alpha)$. This implies that when the preference for environmental quality and the regeneration rate are large, economic growth is sustainable in the long run. This is in line with the literature on endogenous economic growth and the environment (e.g. Smulders (2005), Smulders et al. (2005), Smulders & de Nooij (2003), Acemoglu et al. (2009)). The intuition is the following. When individuals place a large weight on environmental quality, the social demand for clean technologies is high. The high demand for clean technologies combined with large regeneration rate of environmental quality improves environmental quality in equilibrium. As each generation is born richer, with both a higher stock of knowledge and capital, they allocate a higher share of their income to environmental quality. Hence, faster innovation and capital accumulation are associated with cleaner technologies. The equilibrium exhibits positive economic growth and good environmental quality.

In the following, I analyze the properties of the BGP equilibrium when $b\phi < \frac{1}{1+\gamma}$ in proposition 2.

Proposition 2. *1. For any feasible parameter values $(\alpha, \lambda, \gamma, \varphi, \phi)$, there exists a regeneration rate $\bar{b} > 0$ such that, for all $b < \bar{b}$, the equilibrium exhibits degrading environmental quality ($g_E^* < 0$).*

2. *For any feasible parameter values $(\alpha, \lambda, \gamma, \varphi, b)$, there exists a preference for environment $\bar{\phi} > 0$ such that, for all $\phi < \bar{\phi}$, the equilibrium exhibits negative economic growth ($g_Y^* < 0$) and degrading environmental quality ($g_E^* < 0$).*

Proposition 2 summarizes the two main results of the paper which relate intergenerational externalities to unsustainable economic growth. Let us begin by analyzing the equilibrium presented in proposition 2.1. First, for any preference for environmental quality relative to consumption ($\forall \phi$), if regeneration rate is $b < \bar{b}$, the unique BGP equilibrium has deteriorating environmental quality ($g_E^* < 0$). While technological progress and capital accumulation generate economic growth, they also generates pollution, which damages future environmental quality. Consumers can offset pollution using cleaner technologies. However, since there is an intergenerational externality that distorts the demand for clean technologies, the social demand for clean technologies is not sufficiently high to compensate for the negative effect of growth on pollution. Hence, environmental quality degrades in equilibrium. In this equilibrium, economic growth is not sustainable because the intergenerational externality distorts the demand for clean technologies. Individuals do not care about the effect of capital accumulation on future environmental quality. Consequently, their social demand for clean technologies is not high enough to guarantee a non-negative growth rate of environmental quality g_E^*.

The second result in proposition 2 is related to the growth rate of the economy. In proposition 2.2, I present the conditions for negative economic growth $g_Y^* < 0$ in addition to degrading environmental quality. If the weight that individuals place on environmental quality relative to consumption (ϕ) is bounded by $\phi < \bar{\phi}$, where

$$\bar{\phi} = \frac{1}{(1+\gamma)\left((1+b)(1+\varphi\lambda L_R^*)^{\gamma(1-\alpha)} - 1\right)},$$

then, in the BGP equilibrium, environmental quality and output degrade. One may expect that economic growth is fast in economies where agents care little about the environment. However, I show that when agents care little about the environment, environmental quality not only deteriorates but economic growth can be negative. This second result is particularly interesting considering the complementarity in economic growth and environmental quality in equilibrium. This implies that while environmental quality and output could grow simultaneously in equilibrium, this does not happen because the demand for clean technologies is not sufficiently high.

Intuitively, there is fast initial economic growth due to capital accumulation and technological advances. Economic growth generates pollution and rapid environmental degradation. With the preference for environmental quality being low, the social demand for clean technologies is not sufficiently high to stop environmental degradation, which increases the speed of the process. The quality of the environment then becomes low, and the disutility from low environmental quality higher. Therefore, economic growth must slow down, resulting in stagnation. This result emphasizes the stark consequences of intergenerational externalities for sustainable economic growth. Preference for environmental quality is not only important to achieve high environmental quality, but it is also important to maintain positive economic growth.

Equilibrium results here differ from related work on endogenous economic growth and the environment. In the literature, it is common to consider the problem of a social planner who maximizes the utility of every generation. Thus, there are no intergenerational externalities and the social planner chooses the optimal levels of capital, technological progress and clean technologies that guaranteed sustainable growth. With intergenerational externalities, however, investment in capital is high and the demand for clean technologies is low. Thus, along the BGP equilibrium, economic growth and environmental quality improve at a slower rate than the allocation of a social planner. In this paper, I show how intergenerational externalities reduce the possibilities to achieve sustainable economic growth. Next, I turn to the analysis of the social planner and I compare the allocation of the planner to the market outcome.

4 The problem of a social planner

In this section, I compare the market equilibrium established above to the equilibrium allocation of a social planner. A social planner maximizes the utility of all generations choosing the amount of labor allocated into the research sector, capital accumulation and the share of clean technologies, subject to the resource constraint, the stock of environmental quality and technological progress.

Formally, the problem of a social planner:

$$
\begin{aligned}
\max_{L_{R,t},K_{t+1},z_t} \quad & \sum_{t=0}^{\infty} \beta^t \left(U_1(C_t) + \phi U_2(E_t)\right) \\
\text{s.t.} \quad & Y_t = C_t + K_{t+1} - K_t \\
& E_{t+1} = (1+b)E_t - Y_t z_t^{\gamma} \\
& A_{t+1} = A_t \left(1 + \varphi \lambda L_R\right)
\end{aligned}
\tag{19}
$$

where $\beta \in (0,1)$ is the weight the social planner places on the utility of each generation. Environmental quality, the resource constraint and technological progress evolve as before (cf. equations (5), (**??**) and (8)). I sketch the calculation of the optimum in appendix B-1.1.

The creative destruction nature of innovations in a Schumpeterian model introduces an externality. Monopolists do not capture the entire social gain created by an innovation because there is a business stealing effect. Furthermore, a successful innovation destroys the surplus attributable to the previous innovation by making it obsolete.

Since the social planner maximizes the utility of all generations, the Euler equation captures the usual trade-off between consumption among different generations.

$$
\frac{C_{t+1}}{\beta C_t} = \alpha \frac{Y_{t+1}}{K_{t+1}} \left(1 + \frac{1}{1+\gamma}\right) + 1 \tag{20}
$$

Furthermore, the social planner internalizes the negative effect of capital accumulation on pollution. If a generation consumes one less unit, the return from investment is the green return on capital. In the market economy studied in the previous section, individuals receive an inefficiently high return from capital and, hence, they do not have incentives to internalize the effect of capital accumulation on pollution.

Since the social planner takes into consideration the well-being of every generation, clean technologies are chosen taking into account the benefits that clean technologies place on future generations. In particular, it captures the trade-off between current generation's consumption and future generations' environmental quality. Formally, the optimality condition is:

$$\frac{1}{(1+\gamma)z_t^{\gamma}C_t\beta} - \frac{(1+b)}{(1+\gamma)z_{t+1}^{\gamma}C_{t+1}} = \frac{\phi}{E_{t+1}} \tag{21}$$

Notice the difference to the market allocation of clean technologies in the previous section (11). While the choice of clean technologies is an *intra*temporal decision for each generation, the social planner's choice is an *inter*temporal decision. While individuals demand clean technologies given their trade-off between consumption and environmental quality when old, the social planner demands clean technologies given the trade-off between consumption of one generation and environmental quality of the next generation. Therefore, the social planner internalizes the intergenerational externality on environmental quality for future generations.

In the following, I arrange the system of equations that describe the economy and characterize the optimal BGP equilibrium focusing on the growth rates of output and environmental quality:

$$(1+g_E) = \frac{(1+g_Y)^{1+\gamma(1-\alpha)}}{(1+\lambda\varphi L_R^{sp})^{\gamma}}, \tag{22}$$

$$\frac{(1+g_Y-\beta)}{\beta\alpha\left(1+\frac{1}{1+\gamma}\right)}\left(\frac{(b-g_E)\beta\phi(1+\gamma)}{(1+g_E)-\beta(1+b)}-1\right) = \frac{g_Y(b-g_E)\beta\phi(1+\gamma)}{(1+g_E)-\beta(1+b)}. \tag{23}$$

Proposition 3 summarizes the equilibrium BGP.

Proposition 3. *Let $\beta > c(b,\phi,\gamma)$, where the value of the constant $c > 0$ is given by (24). Then, there exists a BGP equilibrium. In the BGP equilibrium environmental quality improves and it reaches the maximum ($E = E^{\max}$, $g_E^* = 0$). Furthermore, there is positive economic growth ($g_Y^* > 0$). The BGP equilibrium is characterized by sustainable economic growth.*

The constant $c(b,\phi,\gamma)$ that guarantees the existence is given by:

$$c(\alpha,b,\phi,\gamma) = \frac{1}{1+b+b\phi\,(1+\gamma)} \tag{24}$$

When the social planner discounts the utility of future generations at a rate higher than 24, there exists an equilibrium which exhibits sustainable economic growth, i.e, economic growth and environmental quality improve. This result is consistent with the literature (e.g. Aghion & Howitt (1998a), Grimaud (1999)). As the social planner maximizes the utility of all generations, it internalizes the intergenerational externalities in innovation and environmental quality. Intuitively, technological progress and capital accumulation drive economic growth. While economic growth generates pollution, the optimal demand for clean technologies offsets the negative effect of pollution on the economy. In the long run, there is a transition from dirty to clean technologies which achieve sustainable economic growth.

5 Model extension with altruistic individuals

The consumers studied thus far care only about their own welfare and leave no bequests. To gain further insight into the intergenerational externalities in growth and the environment, I extend the model to investigate the case in which individuals are altruistic. I consider two specifications. First, I consider individuals who care about the utility of their children. Second, I introduce the possibility to receive and leave bequests in addition to caring about the utility of children.

I assume that parents care about their children's welfare by weighting the children's utility in their own utility function. The utility of an individual born at time t is then given by

$$\max_{s_t, z_t, q_{t+1}} U_1(c_{o,t+1}) + \phi_t U_2(E_{t+1}) + \beta \left(U_1(c_{o,t+2}) + \phi_{t+1} U_2(E_{t+2})\right) \tag{25}$$

where β is the weight on the children's utility, $c_{o,t+2}$ is the consumption of generation $t+1$ in period $t+2$ (children's consumption), and E_{t+2} is environmental quality in period $t+2$ (children's environmental quality). Each individual cares about its own utility and the utility of children, discounted at rate $\beta \in (0,1)$. Although each individual cares directly only about the subsequent generation, these series of intergenerational links imply that each generation implicitly cares about the utility of all future generations. As before, the demand for clean technologies is a social demand while capital investment and bequests are individual decisions.

Bequests are introduced as follows. In addition to wage income, young individuals receive a bequest from their parents in the first period of life. Individuals, in turn, leave bequests to the next generation in the second period, $q_{t+1} \geq 0$. Thus, the modified budget constraints of young and old individuals become:

$$
\begin{aligned}
s_t &= w_t + q_t && (26) \\
c_{o,t+1} + q_{t+1} &= (1 + r_{t+1})s_t + \pi_t && (27)
\end{aligned}
$$

Labor supply, profits and the stock of environmental quality are as in the previous section. Young individuals maximize their lifetime utility and the next generation's utility (25) by demanding clean technology, z_t, saving, s_t, and leaving bequests q_{t+1} subject to budget constraints (26 and 27), environmental constraint (5) at t and $t + 1$, while taking prices, environment and bequests when they are born as given.

As before, the social demand for clean technology reflects the trade-off between income and environmental quality. However, the social demand for clean technology now also accounts for the effect on the utility of the children:

$$
\begin{aligned}
U_1^{'}(c_{o,t+1})\frac{\partial c_{o,t+1}}{\partial \pi_t}\frac{\partial \pi_t}{\partial z_t} - U_2^{'}(E_{t+1})\frac{\partial E_{t+1}}{\partial P_t}\frac{\partial P_t}{\partial z_t} \\
+\beta U_1^{'}(c_{o,t+2})\frac{\partial c_{o,t+2}}{\partial Y_{t+1}}\frac{\partial Y_{t+1}}{\partial z_{t+1}}\frac{\partial z_{t+1}}{\partial z_t} \\
-\beta U_2^{'}(E_{t+2})\left(\frac{\partial E_{t+2}}{\partial E_{t+1}}\frac{\partial E_{t+1}}{\partial P_t}\frac{\partial P_t}{\partial z_t} + \frac{\partial E_{t+2}}{\partial P_{t+1}}\frac{\partial P_{t+1}}{\partial z_{t+1}}\frac{\partial z_{t+1}}{\partial z_t}\right) = 0.
\end{aligned} \tag{28}
$$

The social demand for clean technology represents the socially desirable level of clean technology for each generation. As only one generation consumes and gets utility, governments only consider the utility of this generation when determining the socially desirable level of technological cleanliness. As governments choose z_t through voting, governments are as myopic as individuals and they do not consider the externality on future generations. Therefore, there is no strategic interaction in the choices of clean technologies between different generations, $\frac{\partial z_{t+1}}{\partial z_t} = 0$. The trade-off in (28) reduces to:

$$
U_1^{'}(c_{o,t+1})\frac{\partial c_{o,t+1}}{\partial \pi_t}\frac{\partial \pi_t}{\partial z_t} - U_2^{'}(E_{t+1})\frac{\partial E_{t+1}}{\partial P_t}\frac{\partial P_t}{\partial z_t} - \beta U_2^{'}(E_{t+2})\frac{\partial E_{t+2}}{\partial E_{t+1}}\frac{\partial E_{t+1}}{\partial P_t}\frac{\partial P_t}{\partial z_t} = 0, \tag{29}
$$

where the increase in social demand for dirty technologies have several effects. It increases utility due to higher consumption and it decreases utility due to worse environmental quality for the current and future generation.

As before, substituting monopoly profits (A-4), $s_t = K_{t+1}$ and r_{t+1} into the consumption equation (27) and the equation for the dynamics of environmental quality (5), the social demand for clean technology becomes:

$$\frac{1}{\phi c_{o,t+1}\left(\frac{1}{E_{t+1}} + \frac{\beta(1+b)}{E_{t+2}}\right)} = \frac{(1+\gamma)}{\alpha(1-\alpha)} z_t^{\gamma}, \tag{30}$$

where E_{t+1} is given by (5), consumption is $c_{o,t+1} = K_{t+1} + \alpha^2 Y_{t+1} + \alpha(1-\alpha)Y_t - q_{t+1}$, the term on the left hand side is the marginal rate of substitution, and the right hand side is the marginal rate of transformation. The social demand for clean technology of altruistic individuals (30) is similar to that of non-altruistic individual. The marginal rate of substitution of altruistic individuals (30) incorporates the effect of clean technologies on future environmental quality while the marginal rate of substitution only accounted for the effect of clean technologies on themselves.

Altruistic individuals must decide how much of their income to consume when old and how much to leave for their children. The Euler equation describing the inter-temporal trade-off between one's own consumption and the consumption of the next generation is as follows:

$$U_1'(c_{o,t+1})\frac{\partial c_{o,t+1}}{\partial q_{t+1}} + \beta\left(U_1'(c_{o,t+2})\frac{\partial c_{o,t+2}}{\partial q_{t+1}}\right) = 0. \tag{31}$$

This Euler equation can be rewritten in the usual form:

$$\frac{c_{o,t+2}}{\beta c_{o,t+1}} = 1 + r_{t+2}. \tag{32}$$

The inter-temporal decision considerably changes the equations that characterize equilibrium. As non-altruistic individuals in the previous section do not face any inter-temporal decision on capital investment, there is over-accumulation in capital. However, with altruistic individuals there is an inter-temporal decision on investment which allows capital allocation more efficiently. In the previous section, I show that by only focusing on the intra-temporal decision of individuals, there are intergenerational externalities associated with demand for clean technologies and capital. The social demand for clean technologies was not enough to compensate for the effect of individual capital accumulation on pollution. Now, we consider the case in which the effect of individuals' actions can be internalized through the inter-temporal decisions.

5.1 Individuals do not receive and leave bequests

I begin by considering the corner solution, in which individuals do not leave bequests ($q_t = 0$). The investment decision of individuals is unchanged from the previous section, and, hence, the dynamics of capital is as before, equation (10). Equation (30) describe the social demand for clean technologies with altruism. Now, individuals internalize the aforementioned inter-generational externality on environmental quality. As individuals do not leave or receive bequests, there is no inter-temporal decision regarding consumption. The production side of the economy is the same as the one presented in the previous section, and, hence, the dynamics of innovation do not change. Thus, the economy is described by the dynamic system given by equations (5), (8), (10), and (30). To characterize the BGP, the dynamic system is rewritten in terms of growth rates in the appendix.

Let us interpret the new BGP equilibrium. The social demand for clean technologies (30) is the only equation that changes by introducing altruistic individuals, and, hence, the production side of the economy does not change.

$$\frac{(1+g_E)^2}{(b-g_E)\left((1+g_E)+\beta(1+b)\right)} = \frac{\phi(1+\gamma)}{\alpha(1-\alpha)}\left(\alpha(1-\alpha)(\lambda+1)+\alpha^2(1+g_K)\right) \tag{33}$$

The new term in equation (33), $\frac{(1+g_E)}{(1+g_E)+\beta(1+b)}$, accounts for the effect of the demand for clean technologies on the subsequent generation.

The BGP equilibrium is the g_E and g_K that solves the system of equations (18) and (33). The following proposition describes the conditions for existence and uniqueness of an equilibrium. I present the proof in appendix.

Proposition 4. *Let $\phi < \tilde{\phi}$, where the value of the constants ϕ and $\tilde{\phi}$ are given in (34). Then,* *there exists a unique balanced growth path equilibrium.*

The upper limit on ϕ 34 takes into account the effect of the demand for clean technologies today on the subsequent generation's utility, $\frac{1}{1+\beta(1+b)}$.

$$\tilde{\phi} = \frac{(1-\alpha)}{(1+\widetilde{\beta}(1+b))(1+\gamma)\left((1-\alpha)(\lambda+1)+\alpha(1+\frac{(\lambda\alpha-1)(\varphi-1)}{\alpha})\gamma\right)}. \tag{34}$$

Note that this is the only difference between the conditions for existence with altruistic and non-altruistic individuals (34). The existence condition with altruistic individuals is more restrictive than before, which means that it is harder to get unsustainability. This is because the intergenerational effect of clean technologies on the quality of the environment is internalized. The BGP equilibrium with altruistic individuals is presented in the following.

Proposition 5. *1. For any feasible parameter values $(\alpha, \lambda, \gamma, \varphi, \phi)$, there exists a regeneration rate $\bar{\bar{\tilde{b}}} > 0$, such that for all $\tilde{b} < \bar{\bar{\tilde{b}}}$, the equilibrium exhibits degrading environmental quality ($g_E^* < 0$).*

2. For any feasible parameter values $(\alpha, \lambda, \gamma, \varphi, b)$, there exists a preference for environment $\bar{\bar{\tilde{\phi}}} > 0$, such that for all $\tilde{\phi} < \bar{\bar{\tilde{\phi}}}$, the equilibrium exhibits negative economic growth ($g_Y^* < 0$) and degrading environmental quality ($g_E^* < 0$).

The existence condition is more restrictive with altruistic individuals. However, the equilibrium unsustainability results do not change. The intuition behind these results coincides with that of selfish individuals in the previous section. First, for any preference for environmental quality and a positive regeneration rate $b < \bar{\bar{\tilde{b}}}$, the equilibrium growth rate of environmental quality is always negative. The condition on b is more restrictive than earlier, as $\bar{\bar{\tilde{b}}} < \bar{b}$. Second, if the preference for environmental quality is low, $\phi < \bar{\bar{\tilde{\phi}}}$ where:

$$\bar{\bar{\tilde{\phi}}} = \frac{(1-\alpha)}{(1+\gamma)(1+\beta(1+b))((1-\alpha)\lambda+1)\left(\left(1+\frac{(\lambda\alpha-1)(\varphi-1)}{\alpha}\right)^{\gamma(1-\alpha)}(1+b)-1\right)},$$

and $\bar{\bar{\tilde{\phi}}} < \bar{\phi}$, economic growth and environmental quality degrade in equilibrium.

Even when individuals care about its children, the social demand for clean technologies is too low to internalize the effect of capital accumulation on pollution. This result strengthens the result presented earlier, stating that there is too much investment in capital and too little demand for clean technologies to provide conditions for sustainability. Moreover, this proposition clarifies that caring for the well-being of children is not enough to internalize the intergenerational externality generated from high capital accumulation on pollution.

5.2 Individuals receive and leave bequests

As described in proposition 5, caring for the future generation alters the demand for clean technologies but the capital decisions are unchanged, which results in a BGP equilibrium with unsustainable growth. Let us now explore the interior solution; the situation in which individuals can receive and leave bequests ($q_t > 0$), in addition to taking into account the utility of future generations.

The dynamic behavior of the economy now accounts for the inter-temporal decision of individuals equation (32). The production side of the economy is unchanged. However, consumers' decisions, changes in two ways. First, as individuals care about future generations, the effect of clean technologies on future generations' environmental quality is considered (30). Second, the inter-temporal decision between one's own consumption and that of the future generation is accounted for (32). The two equations that characterize the equilibrium are:

$$(1+g_E) = \left(\frac{(1+g_K)}{(1+\frac{(\lambda\alpha-1)(\varphi-1)}{\alpha})^{\gamma}}\right)^{1-\alpha} \tag{35}$$

$$\frac{(1+g_E)^2}{(b-g_E)\left((1+g_E)+\beta(1+b)\right)} = \frac{\phi(1+\gamma)}{\alpha(1-\alpha)}\left(\alpha^2(1-\lambda)(1+g_K)+\lambda\alpha(1-\alpha)-\frac{\alpha^2\beta(1+g_k)g_k}{(1+g_k)-\beta}\right), \tag{36}$$

where $\lambda\alpha(1-\alpha)(1+g_k) - \frac{\alpha^2\beta(1+g_k)g_k}{(1+g_k)-\beta}$ captures the inter-temporal consumption decision, and the term $\frac{1+g_E}{(1+g_E)+\beta(1+b)}$ includes the effect of the demand for clean technologies today on the subsequent generation.

In proposition 6, I present the condition for existence of an equilibrium. This proposition holds for reasonable parameter values.

Proposition 6. *There exists a BGP equilibrium if and only if*

$$\phi^2 b^2(1+\beta(1+b))^2(1+\gamma)^2(1-\alpha)^2\alpha^2 + (1+\lambda)(-1(\beta-1))\beta^2\alpha - (1+\beta^2(\alpha-1)) \tag{37}$$

$$> 2b(1+\beta(1+b))(1+\gamma)(2(1-\beta)\beta\alpha + (1+\beta)^2(1-\alpha)(1+\lambda)). \tag{38}$$

Then, the BGP equilibrium exhibits economic growth ($g_Y^* > 0$) and improving environmental quality ($g_E^* > 0$).

The decision of individuals (36) involves quadratic terms in g_k and g_E and, therefore, it is no longer an increasing concave curve. The quadratic term in the quality of the environment comes from the next generation's utility in today's maximization decision. The quadratic term in the growth rate of capital comes from the inter-temporal consumption decision.

The quadratic term for environmental quality yields a U shaped curve that crosses the vertical axis in the positive space. The quadratic capital term is an S shaped curve, which crosses the horizontal axis in the negative space. The firms' problem (35) crosses these curves in the upper-right quadrant and in the lower-left quadrant, generating multiple equilibria. Only the equilibrium in the upper-right quadrant is stable, and, hence, there exists a unique stable BGP equilibrium in which both the economy and the environment grow at positive rates.

The intuition behind this equilibrium is the following. Consumers care about the utility of the future generation. Although each generation cares directly only about their children, this series of intergenerational links implies that each generation indirectly cares about the utility of all future generations. Thus, the social demand for clean technologies internalizes the effect of demanding dirty technologies today on future environmental quality. The intergenerational environmental externality is therefore internalized. Furthermore, by receiving and leaving bequests, the intergenerational investment externality is internalized. The consumption decision allows individuals to reduce investment in capital, thereby reducing pollution generated growth.

The introduction of altruism alters the decision of the individual to account for the intergenerational effect of their actions on the economy. By allowing individuals to care about future generations, the social demand for clean technologies increases. Furthermore, when individuals are allowed to leave bequests for their children, investment in capital decreases and internalizes the externality that generated unsustainability in the economy.

6 Conclusion

This paper provides an endogenous growth model with overlapping generations to study the effects of intergenerational externalities on sustainable growth. If current generations do not internalize the effects of their actions on future generations, there is an intergenerational externality that distorts the demand for clean technologies. While the focus is often on the supply of clean technologies, I examine how the demand for clean technologies can be critical for sustainable growth. I find that each generation demands insufficient clean technologies to offset the effect of pollution on future environmental quality. Hence, environmental quality can degrade in equilibrium. Furthermore, in economies where individuals care little about environmental quality, I find that both economic growth and environmental quality might degrade in equilibrium. Overall, examining intergenerational externalities, I focus on another difficulty we face in achieving sustainable growth.

A limitation of this study is that I abstract from the analysis of transitional dynamics. I focus on studying situations that could arise from intergenerational externalities as opposed to how an economy would get to that situation. Hence, I highlight the importance of the interaction between different generations as they can lead an economy to an equilibrium with no economic growth. However, to determine what the equilibrium path would be, one would need to study transitional dynamics.

As I mention above, my focus is on studying the incentives to demand for clean technologies as opposed to the incentives to supply clean technologies. The studies that focus on the supply of clean technologies suggest the need for policy instruments, such as taxes and subsidies, to increase the incentives to supply clean technologies. In line with this, a policy implication of my study is the need to direct policy instruments to promote the demand for clean technologies. We can find examples of similar policies worldwide. For example, many OECD countries subsidize the purchase of energy efficient appliances.[24]

There are several possibilities for selfish individuals to raise their chances to achieve sustainable growth. First, one could consider individuals who care about their children. Second, governments could introduce policies to internalize intergenerational externalities. Finally, intergenerational coordination mechanisms could link different generations to achieve sustainable growth.

The possibilities to achieve sustainable growth may raise when individuals care about their children. When individuals care about the utility of their children, they may demand more clean technologies. More demand for clean technologies could help offset the negative effect of pollution on future environmental quality. However, in my context, it is not clear whether demanding more clean technologies is enough to achieve sustainable growth. Even when individuals care about their children, the saving decision of altruistic individuals is constraint because they must consume all their assets before they die. A possibility to change the saving decision is to consider altruistic individuals who not only care about the utility of their children but also leave bequests. This might be a possibility for finitely lived individuals to achieve sustainable growth.

Finitely lived individuals could also achieve sustainable growth when governments implement optimal policies. The existence and implementation of optimal policies to achieve sustainable growth is the focus of Acemoglu et al. (2009) and many others. They characterize dynamic tax policies as a function of the degree of substitutability between clean and dirty inputs and environmental and resource stocks. There is also an extensive literature analyzing whether environmental taxes yield a double dividend because they reduce pollution and increase growth (e.g., Goulder (1995)). These papers, however, neglect intergenerational externalities. My results suggest that optimal environmental taxation must also be considered within the context of overlapping generations as in Bovenberg & Heijdra (1998). Such environmental policies may be adequate to internalize intergenerational externalities and, therefore, achieve sustainable economic growth.

Finally, one could examine market based intergenerational coordination mechanisms between different generations. Such intergenerational coordination mechanisms would link different generations, and would require later generations to compensate earlier generations for investing in the environment. Rangel (2003) and Von Amsberg (1995) suggest a coordination mechanism that creates obligations for future generations to pay for earlier generation's social security in exchange for investments in environmental quality.

Notes

[1]Investment in clean technologies have increased 60% from 2006 to 2007, and 72% from 2007 to 2008 (UNEP, 2010).

[2]This share includes hydro and non-hydro renewables.

[3]The International Environmental Agency (IEA) defines several generations of renewable energy technologies. First-generation technologies, which are already mature and economically competitive, include biomass, hydroelectricity, geothermal power and heat. Second-generation technologies are market-ready and include solar panels, second generation biofuels, photovoltaics, wind power, solar thermal power stations, and modern forms of bioenergy. There are also cleaner fossil fuel technologies such as carbon capture and storage (CCS).

[4]See Brock & Taylor (2005), Xepapadeas (2006) and Smulders (2000) offer excellent surveys.

[5]At birth, all workers enter the labor market with the knowledge they inherit from their parents' generation.

[6]It is also possible to consider infinite patent life. This substantially complicates the analysis because it requires patent rights enforcement across generations, without adding any new insight. (Griffith et al., 2003) study a Schumpeterian endogenous growth model in overlapping generations and also consider a one period lifetime of patents. See (Chou & Shy, 1993) for the implications of the lifetime of patents in OLG models. They investigate how the duration of patents affects investment in R&D and welfare by using an OLG model of product innovation.

[7]Under current US law, the term of patent is 20 years from the earliest claimed filing date.

[8]While each firm generates a stochastic stream of revenues, the presence of many independent firms implies that each should maximize expected profits.

[9]In this paper I abstract from population growth to avoid explosive growth. Knowledge spillovers with scale effects in the R&D sector drive economic growth. These scale effects combined with population growth would lead to explosive growth. Population growth could be included reducing the scale effects of knowledge spillovers. This could be captured, for instance, changing the production of ideas from $\mu = \lambda L_{R,t}$ to $\mu = \lambda L_{R,t}^{\sigma}$. See Dinopoulos & Thompson (1998) for a Schumpeterian model without scale effects.

[10] At any point in time there will be a distribution of productivity parameters $A_{i,t}$ across the sectors of the economy, with values ranging from 0 to $A_t^{\max}$.

[11] The monopolist does not face potential competition from previous innovators since I assume drastic innovations. Drastic innovations imply that the monopolists extract the full expected net present value of monopoly profits generated by the innovation. While the assumption of drastic innovations might sound extreme, non-drastic innovations do not add additional insights to the paper while they complicate the analysis considerably. See (Denicolo, 2001) for an analysis of economic growth with non-drastic innovations.

[12] This technology index is widely used in the sustainable growth literature. See for example Stokey (1998) and (Aghion & Howitt, 1998b) in the context of sustainable growth.

[13] The size of the population is normalized to one, $L = 1$, and there is no population growth. In this paper, population growth leads to explosive economic growth. While knowledge spillovers with scale effects in the R&D sector drive economic growth, these scale effects combined with population growth would lead to explosive growth. Population growth could be included reducing the scale effects of knowledge spillovers. This could be captured, for instance, changing the production of ideas from $\mu = \lambda L_{R,t}$ to $\mu = \lambda L_{R,t}^{\sigma}$.

[14] This feature is important as it simplifies the calculation of the closed form solution.

[15] As I assume a closed economy, in equilibrium household's assets equal the capital stock.

[16] People consume all their income before dying. If consumers were allowed to leave some of their income unused, the strength of my results could vary.

[17] This result is standard in the OLG literature and holds because individuals are not altruistic.

[18] The regeneration rate b is linear in environmental quality, which implies that natural regeneration is independent of the environment. A common modification is to assume that natural regeneration changes with the stock of the environment. This assumption is common for renewable resources such as forestry or fisheries. While I do not use a general specification, I believe my results are robust as the linear specification is the one for which it is harder to prove my results.

[19] Other papers in the literature employ more detailed descriptions of environmental quality (Smulders (2000) and Xepapadeas (2006)). For example Smulders (2000) describes environmental quality as a Schaefer function. This specification is useful to study the level of environmental quality in equilibrium. The focus on this paper is on long run growth rates, and hence, I consider the specification in equation (5) more

suitable. I employ a similar approach to others in the literature, such as Aghion & Howitt (1998a), John & Pecchenino (1994), John et al. (1995). In contrast to them, I define environmental quality as a positive variable as did Acemoglu et al. (2009). Instead, they define environmental quality as a negative variable where the environment is explained as the difference between the actual level of environmental quality and the optimal level. This transformation and my specification (equation (5)) hold the same properties and, hence, the results are the same. The only difference lies in the interpretation of environmental quality.

[20]For instance, John et al. (1995) consider a short-lived government that internalizes the intragenerational externality by setting a pigovian tax on abatement but ignore the intergenerational externality. Jones & Manuelli (2001) consider individuals who choose by voting for short-lived governments that set environmental policies that only internalize intragenerational externalities. Government failure in regulation is also common in the double dividend literature. In this case, environmental policy is set correctly but other domestic distortions, such as distortions caused by taxes on labour supply, are not considered (Bovenberg & de Mooij (1994), Goulder (1995)).

[21]Grimaud (1999) studies a decentralized economy compatible with Aghion & Howitt (1998a) but as he employs an infinite horizon model, individuals internalize the effect of their actions on future generations.

[22]As long as R&D input, $L^*_{R,t}$, is non-stochastic, the quality $A^{\max}_t$ and total output Y_t are non-stochastic (Aghion & Howitt, 1992).

[23]This is the most common definition of sustainable growth. Another common definition is to consider that an economy is sustainable when utility improves, or at least it does not deteriorate, over time (ref). I choose to define sustainable growth in growth rates, as opposed to utility, because I am interested in learning how intergenerational externalities might affect both economic growth and environmental quality. This approach allows me to learn how selfish generations might decrease the chances of future generations to achieve sustainability as opposed to evaluating their well-being by looking at changes in utility over time. See (Brander, 2007) for a great discussion on different definitions of sustainability.

[24]International Environmental Agency, *"Implementation of the 25 energy efficiency policy recommendations in IEA member countries: recent developments"*.

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Appendix

A-1.1 Capital market

In the production process, capital is only used as an input by intermediate producers implying that all intermediate goods are equal to capital: $\int_0^1 A_i x_i di = K$. Since there are μ monopolistic and $1-\mu$ competitive firms:

$$\begin{aligned}
\int_0^1 A_i x_i di &= K \\
\mu A^m x^m + (1-\mu)\int_0^1 A_{i,t} x_{i,t}^c d_i &= K \\
A(x^c)\left(\mu \frac{A^{\max}}{A}\frac{x^m}{x^c} + (1-\mu)\right) &= K
\end{aligned}$$

Let us define $a = \frac{A^{\max}}{A}$ as the productivity improvement of the new technologies in every period and $\frac{x^m}{x^c} = \alpha^{\frac{1}{1-\alpha}}$ is the share of monopolistic and competitive goods in the final good sector. Then,

$$A(x^c)\left(\mu a \alpha^{\frac{1}{1-\alpha}} + (1-\mu)\right) = K$$

Next, let us define $\tilde{\mu} = \mu a \alpha^{\frac{1}{1-\alpha}} + (1-\mu)$. And total capital is:

$$A(x^c)\tilde{\mu} = K$$

We can rewrite total capital to specify the demand for intermediate goods produced by competitive firms:

$$x^c = \frac{K}{A\tilde{\mu}} \tag{A-1}$$

The price of capital for competitive firms is $p_{i,t}^c = rA_i$, from (2.1.2). Since the demand for competitive intermediate goods is (A-1) and taking the inverse demand function into account, the rental price of capital is:

$$
\begin{aligned}
rA_i &= zL_Y^{1-\alpha} A_i \alpha \left(\frac{K}{\tilde{\mu}A}\right)^{\alpha-1} \\
r &= z\alpha \left(\frac{\tilde{\mu}AL_Y}{K}\right)^{1-\alpha}
\end{aligned} \tag{A-2}
$$

A-1.2 Final output market

From the final output market in 1, we can rewrite total output as:

$$
\begin{aligned}
Y_t &= L_{Y,t}^{1-\alpha} z_t \int_0^1 A_{i,t} x_{i,t}^{\alpha} d_i \\
&= L_{Y,t}^{1-\alpha} z_t \left(\mu A^m x^m + (1-\mu) \int_0^1 A_{i,t} x_{i,t}^{c} d_i\right) \\
&= L_{Y,t}^{1-\alpha} z_t \left(A(x^c)^{\alpha} \left(\mu \frac{A^{\max}}{A} \left(\frac{x^m}{x^c}\right)^{\alpha}\right) + (1-\mu)\right)
\end{aligned}
$$

Let us $\tilde{\tilde{\mu}} = \mu a \alpha^{\frac{\alpha}{1-\alpha}} + (1-\mu)$ and substitute the demand for intermediate goods (A-1). Then,

$$
Y = z(\tilde{\mu}AL_Y)^{1-\alpha} K^{\alpha} \left(\frac{\tilde{\tilde{\mu}}}{\tilde{\mu}}\right) \tag{A-3}
$$

A-1.3 Monopoly profits

Monopoly profits, $\pi_i = p^m x^m - rA^{\max} x^m$, represent the value of blueprints in the research sector. The price of intermediate goods for monopolists is $p^m = \frac{rA^{\max}}{\alpha}$. Then,

$$\begin{aligned}
\pi_i &= p^m x^m - rA^{\max} x^m \\
&= x^m A^{\max} x^m \frac{(1-\alpha)}{\alpha} \\
&= x^m A^{\max} x^m \frac{(1-\alpha)}{\alpha} \frac{\mu}{\mu} \frac{K}{\int_0^1 A_i x_i di} \\
&= x^m A^{\max} x^m \frac{(1-\alpha)}{\alpha} \frac{K}{A(x^c)\left(\mu \frac{A^{\max}}{A} \frac{x^m}{x^c} + (1-\mu)\right)} \\
&= \frac{(1-\alpha)}{\alpha} \frac{rK}{\mu} \frac{\frac{\mu A^{\max} x^m}{Ax^c}}{\frac{\mu A^{\max} x^m}{Ax^c} + (1-\mu)} \\
&= \frac{(1-\alpha)}{\alpha} \frac{rK}{\mu} \frac{\mu a \alpha^{\frac{1}{1-\alpha}}}{\mu a \alpha^{\frac{1}{1-\alpha}} + (1-\mu)} \\
&= \frac{(1-\alpha)}{\alpha} \frac{rKa\alpha^{\frac{1}{1-\alpha}}}{\tilde{\mu}} \qquad \text{(A-4)}
\end{aligned}$$

A-1.4 Labor market clearing

From the final sector and R&D firms (2.1.3) and (2.1.1), we obtain the cost of hiring labor in both sectors:

$$w_t^Y = (1-\alpha)\frac{Y_t}{L_{Y,t}} \qquad \text{(A-5)}$$

$$w_t^R = \lambda \pi_{i,t} \qquad \text{(A-6)}$$

Labor market clearing requires wages to equate in equilibrium, $w_t^Y = w_t^R$. From (A-2) we have:

$$\begin{aligned}
(1-\alpha)\frac{Y_t}{L_{Y,t}} &= \lambda\frac{1-\alpha}{\alpha}rK\frac{a\alpha^{\frac{1}{1-\alpha}}}{\tilde{\mu}} \\
\frac{\alpha Y\tilde{\mu}}{rK} &= \lambda L_Y a\alpha^{\frac{1}{1-\alpha}} \\
\mu a\alpha^{\frac{\alpha}{1-\alpha}} + 1 - \mu &= \lambda L_Y a\alpha^{\frac{1}{1-\alpha}} \\
\mu a\alpha^{\frac{\alpha}{1-\alpha}} + 1 - \mu &= \lambda(1-L_R)a\alpha^{\frac{1}{1-\alpha}} \\
\lambda L_R\left(a\alpha^{\frac{\alpha}{1-\alpha}} - 1 + a\alpha^{\frac{1}{1-\alpha}}\right) &= \lambda a\alpha^{\frac{1}{1-\alpha}} - 1
\end{aligned}$$

Then, the number of workers allocated to research and manufacturing sector in equilibrium are:

$$L_R^* = \frac{\lambda\alpha^{\frac{1}{1-\alpha}}a - 1}{\lambda((1+\frac{1}{\alpha})\alpha^{\frac{1}{1-\alpha}}a - 1)}, \tag{A-7}$$

$$L_y^* = 1 - L_R^*. \tag{A-8}$$

A-1.5 The social demand for clean technologies

The demand for clean technologies captures the individual trade-off between consumption and environmental quality established by:

$$U_1'(c_{o,t+1})\frac{\partial c_{o,t+1}}{\partial z_t} + U_2'(E_{t+1})\frac{\partial E_{t+1}}{\partial P_t}\frac{\partial P_t}{\partial z_t} = 0. \tag{A-9}$$

As individuals only consume when old, (A-9) represents an intratemporal decision between consuming goods and environmental quality. An increase in social demand for dirty technologies today increases consumption through higher wages and profits while environmental quality deteriorates. Likewise, demand for cleaner technologies when young, decreases consumption when old while environmental quality improves. In equilibrium, the social demand for clean technologies ensures the balance between private consumption and environmental quality.

Young agents take the price of capital when old as given because the price is determined in the following generation. They also take capital and innovation when young as given because it is predetermined in the previous period. Consumption when old is given by wages (A-5) and total output (A-3), $c_{t+1} = (1+r_{t+1})(1-\alpha)z_t\frac{\tilde{Y}_t}{L_Y}$, where $\tilde{Y}_t = \frac{Y_t}{z_t}$, and environmental quality by (5). The impact of dirtier technologies in consumption and environmental quality is:

$$\begin{aligned} \frac{\partial c_{t+1}}{\partial z_t} &= (1+r_{t+1})(1-\alpha)\frac{\tilde{Y}_t}{L_Y} \\ \frac{\partial R_{t+1}}{\partial z_t} &= -(1+\gamma)z_t^{\gamma}\tilde{Y}_t \end{aligned}$$

The intratemporal decision of individuals (A-9) is:

$$\begin{aligned} \frac{E_{t+1}}{\phi c_{t+1}} &= \frac{-(1+\gamma)z_t^{\gamma}\tilde{Y}_t}{(1+r_{t+1})(1-\alpha)\frac{\tilde{Y}_t}{L_Y}} \\ E_{t+1} &= \phi(1+\gamma)z_t^{\gamma}Y_t \end{aligned}$$

We can rewrite the equilibrium demand for clean technologies as:

$$z_t^{\gamma} = \left(\frac{1+b}{1+\phi(1+\gamma)}\right)\frac{E_t}{Y_t} \tag{A-10}$$

A-1.6 Equations characterizing the balanced growth path

The economy is described by the dynamic behavior of innovation, environmental quality, demand for clean of technologies and capital ((8), 5), (12 and (10)).

$$A_{t+1} = A_t\left(1+\frac{\lambda\alpha^{\frac{1}{1-\alpha}}a_t-1}{(1+\frac{1}{\alpha})\alpha^{\frac{1}{1-\alpha}}a_t-1}(a_t-1)\right) \tag{A-11}$$

$$E_{t+1} = (1+b)E_t - Y_t z_t^{\gamma} \tag{A-12}$$

$$z_t^{\gamma} = \left(\frac{1+b}{1+\phi(1+\gamma)}\right)\frac{E_t}{Y_t} \tag{A-13}$$

$$K_{t+1} = (1-\alpha)\frac{Y_t}{L_Y} \tag{A-14}$$

Balanced growth path (BGP), where variables grow at constant rates, is assumed to describe the equilibrium behavior of the economy. The BGP consists of the growth rates of innovation, environmental quality, clean technologies and capital.

As the number of workers allocated into the research sector is constant over time, the growth rate of technology, g_A, is constant. Let us substitute $a = 1 + \varphi$ to describe the growth rate of technology, $1 + g_A = 1 + \varphi\mu$:

$$1 + g_A = 1 + \varphi \left(\frac{\lambda\tilde{\alpha}(1+\varphi) - 1}{\lambda((1+\frac{1}{\alpha})\tilde{\alpha}(1+\varphi) - 1)} \right) \tag{A-15}$$

where $\tilde{\alpha} = \alpha^{\frac{1}{1-\alpha}}$.

The growth rate of environmental quality is $g_E = b - \frac{Y_t z_t^{\gamma}}{E_t}$. A constant growth rate implies the following identity along the BGP.

$$1 + g_E = (1 + g_Y)(1 + g_z)^{\gamma} \tag{A-16}$$

Finally, the demand for clean technologies in growth rates:

$$(1 + g_K)^{\frac{1+(1-\alpha)\gamma}{1+\gamma}} = (1 + g_E)^{\frac{1}{1+\gamma}} (1 + g_A)^{\frac{(1\alpha)\gamma}{1+\gamma}} \tag{A-17}$$

Finally, (A-14) captures the relationship between capital and output. A constant growth rate implies a constant capital-output ratio, $\frac{Y_t}{K_t}$, which indicates that capital and output grow at the same constant rate along the BGP. The growth rate of the production function and $g_K = g_Y$ determine the growth rate:

$$1 + g_z = \left(\frac{1 + g_K}{1 + g_A} \right)^{1-\alpha}. \tag{A-18}$$

The BGP is characterized by the growth rate of innovation, environmental quality, clean technologies and capital (A-15), A-16), (A-17) and (A-18).

B-1 The social planner's problem

B-1.1 The social planner's maximization problem

The social planner maximizes the utility of all generations (eq. 19). The control variables are the number of workers in the research sector, consumption and clean technologies, and the state variables are technology, capital and environmental quality. The Lagrangian of the optimal growth path is

$$L = \sum_{t=0}^{\infty} \beta^t \left(U(C_t, E_t) + \sigma \left(-K_{t+1} + Y_t - C_t + K_t \right) + \delta \left(-A_{t+1} + A_t \left(1 + \lambda \varphi L_R \right) \right) \right.$$
$$\left. + \xi \left(-E_{t+1} + (1+b)E_t - Y_t z_t^{\gamma} \right) \right). \tag{B-1}$$

First order conditions characterizing the optimal path are presented in what follows. The first-order conditions are presented in general and specific forms.

I begin with the optimality condition for consumption, which states that the marginal utility of consumption must equal the shadow price of capital:

$$\frac{\partial U(C_t, E_t)}{\partial C_t} = \sigma$$
$$\frac{1}{C_t} = \sigma \tag{B-2}$$
$$\tag{B-3}$$

The next optimality conditions is for the number of workers in R&D (L_R), which determines the optimal speed of innovation.

$$\frac{\partial K_{t+1}}{\partial L_R} + \frac{\partial A_{t+1}}{\partial L_R} - \frac{\partial E_{t+1}}{\partial Y_t}\frac{\partial Y_t}{\partial L_R} = 0$$
$$(1-\alpha)\frac{Y_t}{1-L_R}\sigma(1 - \frac{1}{1+\gamma}) - \delta A_t \lambda \varphi = 0 \tag{B-4}$$

where σ and δ are the shadow prices of capital and technology, respectively. Equation (B-4) presents the overall effect of hiring one more worker in the innovation sector relative to the manufacturing sector on the economy.

The first-order condition for investment in technological progress (A) shows the long-run effect of investing one more unit in technological progress:

$$\frac{\partial L}{\partial A_{t+1}} + \beta^{t+1}\left(\frac{\partial Y_t}{\partial A_t} + \frac{\partial A_{t+1}}{\partial A_t} + \frac{\partial E_{t+1}}{\partial Y_t}\frac{\partial Y_t}{\partial A_t}\right)_{t+1} = 0 \quad \text{(B-5)}$$

$$\delta - \beta\left(\delta^{t+1}(1+\varphi\lambda L_R) + \frac{Y_{t+1}}{A_{t+1}}(1-\alpha)\left(\sigma^{t+1} - \xi^{t+1} z_{t+1}^{\gamma}\right)\right) = 0 \quad \text{(B-6)}$$

where ξ is the shadow price of the environment. The combination of the equations (B-4) and (B-6) clears the labor market for the optimal number of workers the social planner would allocate to each sector.

The optimal capital investment is given by the following condition:

$$\sigma = \beta^{t+1}\left(\frac{\partial Y_t}{\partial K_t} + \frac{\partial K_{t+1}}{\partial K_t} + \frac{\partial E_{t+1}}{\partial Y_t}\frac{\partial Y_t}{\partial K_t}\right)_{t+1}$$

$$\sigma = \beta\sigma^{t+1}\left(1 + \alpha\frac{Y_{t+1}}{K_{t+1}}(1 + \xi^{t+1} z_{t+1}^{\gamma})\right) \quad \text{(B-7)}$$

The optimality condition for the demand for clean technologies (z):

$$\frac{\partial K_{t+1}}{\partial z_t} - \frac{\partial E_{t+1}}{\partial z_t} = 0$$

$$\sigma\frac{Y_t}{z_t} - \xi(1+\gamma)\frac{Y_t}{z_t}z_t^{\gamma} = 0 \quad \text{(B-8)}$$

The optimality condition for environmental quality (E):

$$\frac{\partial L}{\partial E_{t+1}} + \beta^{t+1}\left(\frac{\partial E_{t+1}}{\partial E_t} + \frac{\partial U(C_t, E_t)}{\partial E_t}\right)_{t+1} = 0 \quad \text{(B-9)}$$

$$\xi - \beta\left(\xi^{t+1}(1+b) + \frac{\phi}{E_{t+1}}\right) = 0 \quad \text{(B-10)}$$

First-order conditions for shadow prices:

$$K_{t+1} = Y_t - C_t + K_t \quad \text{(B-11)}$$

$$A_{t+1} = A_t\left(1 + \lambda\varphi L_R\right) \quad \text{(B-12)}$$

$$E_{t+1} = (1+b)\,E_t - Y_t z_t^{\gamma} \quad \text{(B-13)}$$

Finally, output must satisfy the following condition:

$$Y_t = A_t L_{Y_t}^{1-\alpha} K^{\alpha} z \tag{B-14}$$

B-1.2 Equation 22

Next I calculate the relationship between economic growth and environmental quality in the supply side of the economy. The growth rate of environmental quality is $g_E = b - \frac{Y_t z_t^{\gamma}}{E_t}$. A constant growth rate implies the following identity along the BGP.

$$(1 + g_E) = (1 + g_Y)(1 + g_z)^{\gamma} \tag{B-15}$$

As the number of workers allocated into the research sector is constant over time, the growth rate of technology, $g_A = \lambda \varphi L_R^{sp}$, is also constant.

$$g_A = \frac{\beta(1 - \alpha + \varphi\lambda) - (1 - \alpha)}{1 - \alpha(1 - \beta)} \tag{B-16}$$

The production function, $Y_t = x_t^{\alpha}(A_t L_{Y,t})^{1-\alpha} z_t$, its growth rate $(1+g_Y) = (1+g_A)^{1-\alpha}(1+g_K)^{\alpha}(1+g_z)$ and the identity $g_K = g_Y$, determine the following growth rate.

$$g_z = \left(\frac{1 + g_K}{1 + g_A}\right)^{1-\alpha} - 1 \tag{B-17}$$

By combining equations (B-15), (B-16) and (B-17), I describe the behavior of the supply side of the economy in the BGP:

$$1 + g_E = \left(\frac{1 + g_K}{(1 + \frac{\beta(1-\alpha+\varphi\lambda)-(1-\alpha)}{1-\alpha(1-\beta)})^{\gamma}}\right)^{(1-\alpha)} \tag{B-18}$$

B-1.3 Equation 23

Optimal consumption is determined by the following equations:

$$U_1 = \sigma \tag{B-19}$$

$$\frac{U_1^t}{\beta U_1^{t+1}} = \alpha \frac{Y_t}{K_t}\left(1+\frac{1}{1+\gamma}\right)+1, \tag{B-20}$$

where the following notation is used $U_1^t \equiv \frac{\partial U(C_t,E_t)}{\partial C_t}$.

I assume log utility, which implies:

$$\frac{\partial U(C,E)}{\partial C} = \frac{1}{C}. \tag{B-21}$$

Based on this, the optimality conditions can be rewritten as follows:

$$\frac{C_{t+1}}{\beta C_t} = \alpha \frac{Y_t}{K_t}\left(1+\frac{1}{1+\gamma}\right)+1 \tag{B-22}$$

$$(1+g_c) = \beta\left(\alpha \frac{Y}{K}\left(1+\frac{1}{1+\gamma}\right)+1\right) \tag{B-23}$$

$$\frac{Y}{K} = \frac{1+g_C-\beta}{\alpha\beta\left(1+\frac{1}{1+\gamma}\right)} \tag{B-24}$$

The optimality conditions for environmental quality are:

$$\xi = \frac{U_1}{(1+\gamma)\,z^{\gamma}} \tag{B-25}$$

$$\frac{U_1}{(1+\gamma)\,z^{\gamma}} = \beta\left(U_2+\frac{U_1^{t+1}}{(1+\gamma)\,z_{t+1}^{\gamma}}(1+b)\right) \tag{B-26}$$

Assuming log utility gives the following:

$$\frac{1}{(1+\gamma)z^{\gamma}C\beta}-\frac{1+\beta}{(1+\gamma)z_{t+1}^{\gamma}C_{t+1}} = \frac{\phi}{E_{t+1}} \tag{B-27}$$

$$\frac{1}{(1+\gamma)z^{\gamma}C\beta}-\frac{1+\beta}{(1+\gamma)\left((1+g_z)\,z\right)^{\gamma}(1+g_C)\,C} = \frac{\phi}{(1+g_e)\,E} \tag{B-28}$$

(B-29)

From the growth rate of environmental quality (E):

$$(1+g_C)\,(1+g_Z)^{\gamma} = (1+g_E) \tag{B-30}$$

$$\frac{1}{(1+\gamma)Cz^{\gamma}}\left(\frac{1}{\beta}-\frac{1+\beta}{1+g_e}\right) = \frac{\phi}{(1+g_e)\,E} \tag{B-31}$$

$$\frac{Y_C}{(1+\gamma)z^{\gamma}}\left(\frac{(1+g_e)-\beta\,(1+\beta)}{\beta}\right) = \frac{\phi}{E} \tag{B-32}$$

$$(1+g_e)-\beta\,(1+\beta) = \frac{\beta\phi}{E}\,(1+\gamma)\,Z^{\gamma}C \tag{B-33}$$

$$\tag{B-34}$$

(from $g_e = b - \frac{Yz^{\gamma}}{E} \;\Rightarrow\; \frac{z^{\gamma}}{E} = \frac{b-g_E}{Y}$)

$$(1+g_e)-\beta\,(1+\beta) = \beta\phi\,(1+\gamma)\,\frac{C}{Y}\,(b-g_E) \tag{B-35}$$

$$\frac{Y}{C} = \frac{(b-g_E)\,\beta\phi\,(1+\gamma)}{(1+g_e)-\beta\,(1+\beta)} \tag{B-36}$$

Combine (B-35) and (B-36) and production function $Y = g_K K + C$:

$$\frac{Y}{K} = \frac{g_K\frac{(b-g_E)\beta\phi(1+\gamma)}{(1+g_E)-\beta(1+b)}}{\frac{(b-g_E)\beta\phi(1+\gamma)}{(1+g_E)-\beta(1+b)}-1} \tag{B-37}$$

Combining both and $g_K = g_C = g_Y$:

$$\frac{1+g_K-\beta}{\alpha\beta\left(1+\frac{1}{1+\gamma}\right)} = \frac{g_K\frac{(b-g_E)\beta\phi(1+\gamma)}{(1+g_E)-\beta(1+b)}}{\frac{(b-g_E)\beta\phi(1+\gamma)}{(1+g_E)-\beta(1+b)}-1} \tag{B-38}$$

$$\frac{1+g_K-\beta}{\alpha\beta\left(1+\frac{1}{1+\gamma}\right)}\left(\frac{(b-g_E)\beta\phi(1+\gamma)}{(1+g_E)-\beta(1+b)}-1\right) = g_K\frac{(b-g_E)\beta\phi(1+\gamma)}{(1+g_E)-\beta(1+b)} \tag{B-39}$$

The equilibrium BGP is the solution in g_E and g_K of the system of equations (22) and (23).

Equation (22) is an upward slopping convex curve that always crosses the horizontal axis in the positive side and the vertical axis in the negative side. And equation (23) is a downward slopping concave curve. There is a unique solution to the system of equations when the curve (23) crosses the vertical axis above the (22) curve. This holds when the growth rate of environmental quality is positive when the economic growth is zero:

$$g_e = \frac{\beta\left(1 + b + b\phi(1+\gamma)\right) - 1}{1 + \beta(1+\gamma)\phi} > 0 \tag{B-40}$$

The term in the denominator is always positive. Hence, the inequality holds if the term in the numerator is also positive, which requires that the following is satisfied:

$$\beta > \frac{1}{1 + b + b\phi\left(1+\gamma\right)} \tag{B-41}$$