

Pesticide use and wildlife loss: a bioeconomic model on the optimal pesticide policy

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Abstract

We analyze the optimal pesticide policy when pesticides released from farms harmfully affect wildlife. We develop a dynamic model that incorporates the change in farmers' welfare and the social benefits when a pesticide policy is implemented. We derive the optimal pesticide policy using a bioeconomic model that gives account of the trade-offs between agricultural production and wildlife. The analysis shows that the pesticide policy depends highly on the economic range in which the agricultural is a profitable activity, as well as the pesticide stock threshold that drives wildlife to extinction. The economic instrument to implement the policy is a discrete tax. We deal in the last section with the tax level and the timing of such policy.

Key words: *Bioeconomics, pesticides, wildlife, taxation, optimal control*

1 Introduction

Environmental policies addressed to internalize the external costs of the harmful effects of pesticide use on human health, groundwater pollution and biodiversity have been widely studied during the last decades. Economic assessments of pesticide control policies have traditionally relied on pure efficiency effects, either comparing the economic costs and environmental benefits, or comparing costs relative to those of alternative control policies (Hoel & Karp, 2002). Such analyses have focused upon a static system where it is implicitly assumed a stationary state (Brethour & Weersink, 2001; Cuyno *et al.*, 2001).

The study of the system dynamics turns out to be important for a number of reasons. The released pesticides last long in the environment, so their harmful effects to wildlife follow a dynamic path as well. Thus, if the regulator wishes to optimally manage the pesticide use in order to maximize the social benefits, she should inevitably tackle the dynamics of pesticide diffusion across the habitat and its dynamical effects on wildlife population.

Although optimal control models of wildlife are not new, earlier models have essentially focused on investigating the optimal wildlife population and the optimal harvesting rate within the context of multispecies and prey-predator models (Kassar *et al.*, 2004; Keith *et al.*, 1985; Omer *et al.*, 2007; Rollins *et al.*, 2004; Skonhofs, 2005).

The main contribution of this paper is the incorporation of an explicit damage function relating pesticide concentration and wildlife population, *i.e.*, we analyze the optimal pesticide policy when pesticides applied in farms harmfully affect wildlife population. We develop a model that incorporates the change in private agricultural benefits following from controlling pesticide use and the social benefits of using less pesticide in agricultural production. We derive the optimal pesticide policy using a bioeconomic model that gives account of the change in farmers' benefits and the non-use benefits of an increasing wildlife population.

Our analysis focuses on specifying a function that explicitly takes into account the damages to wildlife population, instead of a general damage function that usually just measures economic damages (Ko *et al.*, 1992). This analysis allows specifying the biological thresholds, which turn out to be decisive in defining the optimal pesticide control (Clark *et al.*, 1979; Cropper *et al.*, 1979; Swanson, 1994).

It will be shown that, when the initial pesticide concentration across the habitat is higher than the equilibrium concentration, the optimal pesticide policy consists of 'highly' reducing pesticide at the outset, and then gradually increasing pesticide application at decreasing rate. On the other hand, if the initial concentration smaller than the steady state equilibrium, the optimal pesticide use is to slightly decrease its use at a decreasing rate until the equilibrium point. Our analysis shows that along the optimal path the optimal pesticide policy is mainly determined by the pesticide demand, the lifetime of pesticides and their instantaneous harmfulness and the wildlife growth.

As in many cases it is not possible to implement a dynamic tax, a second-best solution is proposed to implement the policy. The second part of this paper introduces

the implementation of discrete taxes. We will show that a system of phasing in a tax relies highly on the initial pesticide concentration and on the number of tax switching.

2 The model

2.1 Statement of the problem

We study two fundamental aspects of controlling pesticide use: a) the change in private benefits following from regulating pesticide use, and b) the social non-use benefits of wildlife, which mainly accrue to society as the number of wildlife species.

Consider a situation in which the farmers use pesticides to increase the potential agricultural production. Pesticides diffuse to neighboring habitats and enter the wildlife food chain, through soil, water, air and biota. Hence, pesticide applied to control a target pest in farms incidentally reduces populations of some wildlife species as well. However, from a social point of view, the harmful consequences of pesticides to wildlife decrease social welfare by diminishing wildlife population as a consequence of increasing mortality, for instance.

The instantaneous welfare W depends thus on benefits derived from the farmer's benefits $Y(x)$ and social non-use benefits from wildlife, $B(L)$; where x represents the use of pesticides and L the wildlife population.

$$W(t) = W(Y(x(t)), B(L(t))) \quad (2.1)$$

We consider social welfare to be additively separable in all its arguments.

The instantaneous farmers' benefits depend basically upon the input allocations. The benefits function, $Y(x)$, which represents the net benefits from cropping at time t , is a strictly concave function in the relevant economic range. Thus, the optimal amount of pesticide application for given prices is given by the maximization of private benefits:

$$\max_{\{x\}} Y(x) \quad (2.2)$$

whose first order condition is given by $Y'(x^*) = 0$ at a some maximizing level, x^* , and the sufficient condition $Y''(x^*) < 0$. Hence, $x^*(w)$ denotes the optimal pesticide demand function, which gives rise to the highest private benefits, $Y^{max}(x^*(w))$, as a function of the unregulated pesticide price, w . For simplicity in notation, we will simply write x and $Y(x)$ hereafter.

In addition to agricultural production, farms yield pesticides which accumulate in the habitat. The flow of pesticides can be represented by $\alpha x(t)$ where α translates pesticide demand into residuals emitted into the wildlife habitat. This parameter is directly related with the property of affinity. Likewise, the pesticide decays as a proportion of the pesticide concentration $\beta\Omega$, where the coefficient β , related to persistence, represents the rate of decay of the pesticide stock. The instantaneous net accumulation of pesticides through the wildlife habitat is then given by $\alpha x(t) - \beta\Omega$.

On the other hand, we consider the social non-use benefits derived from wildlife population. Such activities can include non-consumptive tourism, wildlife viewing, stewardship or the option value¹. For the purpose of this paper, off-farm wildlife is conceptualized as the non-targeted species living in the neighborhood of farms, which is harmfully affected by pesticides. Moreover, it never becomes a pest and thus it does not affect agricultural output. That is, the sole presence of wildlife provides benefits and then wildlife neither increases nor diminishes income. We assume that wildlife population L can be represented as a single species or biomass at a specific point in time. With this aim, let function $B(L)$ represent the values that individuals capture. The benefits are assumed to be increasing at a diminishing rate as population increases:

$$\frac{\partial B}{\partial L} > 0, \quad \frac{d^2 B}{dL^2} < 0 \quad \text{and} \quad \frac{\partial B(0)}{\partial L} = \infty \quad (2.3)$$

The dynamics of wildlife population is described by the function $\rho(L)$, which represents the population growth function. We assume that $\rho(L)$ is strictly concave in L , $\rho(0) = \rho(K) = 0$, $\rho'(L)$ is monotonically decreasing and $\rho''(L) < 0$. Likewise, the pesticides exert an increasing damage, which is captured by $R(\Omega, L)$; it is a concentration-response function, which is expressed in aggregated wildlife mortality rate as a consequence of the exposure to pesticides. We assume that $R(\Omega, L)$ is increasing at a decreasing rate with respect to pesticide concentration, *i.e.* $R_L(0, L) = 0$, $R_L(\Omega, L) > 0$, $R_\Omega(\Omega, L) > 0$ and $R_{\Omega\Omega}(\Omega, L) \leq 0$. Likewise, we assume that it is separable and it can be specified as $R(\Omega) \cdot L$.

2.2 The optimal control problem

The model defines the control strategy that maximizes the present value of society's welfare. The pesticide control problem facing the social planner is simply to choose the time path for pesticide application that optimizes social benefits. Hence the social planner's pesticide management problem is to maximize the discounted net social benefits, *i.e.*, the agricultural private benefits and the non-use benefits of wildlife over time subject to wildlife population growth and the dynamics of pesticide concentration in the habitat:

$$\max_{x(t)} \int_0^{\infty} \{Y(x(t)) + B(L(t))\} e^{-\delta t} dt \quad (2.4)$$

subject to

$$\frac{\partial L(t)}{\partial t} = \rho(L(t)) - R(\Omega(t))L(t) \quad (2.5)$$

$$\frac{\partial \Omega(t)}{\partial t} = ax(t) - \beta\Omega(t) \quad (\alpha, \beta > 0) \quad (2.6)$$

$$L(0) = L_0, \quad L(t) \geq 0 \quad (2.7a)$$

¹ Option value does not indeed depend on wildlife population, but it is part of the total wildlife value.

$$L(t) = 0, \forall t > t_e \text{ where } \Omega(t_e) \geq \Omega_{ext} \quad (2.7b)$$

$$\Omega(0) = \Omega_0, \Omega(t) \geq 0 \quad (2.7c)$$

$$x(0) = x_0, x(t) \geq 0 \quad (2.7d)$$

$$\delta, \mu, \alpha \text{ and } \beta \in \mathbb{R}^+ \quad (2.7e)$$

and the transversality conditions for the infinite-horizon:

$$\lim_{t \rightarrow \infty} e^{-\delta t} \lambda_0(t) = 0, \quad \lim_{t \rightarrow \infty} e^{-\delta t} \lambda_1(t) = 0 \quad (2.8)$$

where

$x(t)$:	Pesticide demand
$Y(x)$:	Total farmers' private benefits
$B(L)$:	Total non-use benefits of wildlife affected by pesticides
$L(t)$:	Wildlife population
$\rho(L)$:	Natural wildlife population growth
$R(\Omega)$:	Wildlife population mortality
$\Omega(t)$:	Average concentration of pesticide across the habitat
α :	Pesticide flow into the habitat coefficient
β :	Decay coefficient
δ :	The social rate of discount

Note that, unlike Gordon-Schaefer model (Clark, 1976) where benefits directly depends on the resource flow, the objective function in equation (2.4) depends on farmers' private benefits and social non-use benefits of wildlife. The pesticide management problem is thus to determine the time path of pesticide use that optimizes net social benefits.

Condition (2.7b) is particularly important when delimiting the problem. It represents that the historical pesticide application has followed a pattern which has driven wildlife to extinction when the pesticide stock is higher than a determined level, Ω_{ext} . That is, once exceeded such limit, it is no possible to have a positive wildlife population.

3 Generalized functional specifications

The solution for this general case is too complex to reach some intuitive conclusions. For ease of exposition, we make use of some familiar functional forms for wildlife population growth (logistic), non-use benefits (logarithmic) and the concentration-response function (linear) so as to shed some light on the problem.

3.1 Statement of the problem

Consider representative farmers, making input decisions based in maximizing profits. From this point of view, pesticide application decisions are based on the price of the regulated agricultural pesticide, w ; prices of other inputs; and output price. In this study, we approach the problem within a damage control framework for the agricultural system, originally suggested by Lichtenberg & Zilberman (1986). We assume that the pesticide contribution to increase the agricultural output can be modeled as a damage abatement input rather than a production input. Such damage control inputs do not increase potential output but rather they may decrease it.

Furthermore, other productive inputs and prices are assumed to be constants, which determine the maximal potential output, A . Thus, the benefits function can be considered as the profits obtained from agricultural production, with pesticides as the main input. Likewise, we assume constant marginal costs of applying pesticide, *i.e.*, cost linearly depends on the amount of pesticide applied:

$$Y(x) = AG(x) - wx \quad (3.1)$$

where A is the maximum potential agricultural production; this parameter captures price and production technology as well. Lichtenberg & Zilberman (1986) define the function $G(x)$ as the pesticide effectiveness to kill pests. Such function possesses the properties of a cumulative probability distribution: $G(0) = 0$, $\lim_{x \rightarrow \infty} G(x) = 1$, and $G'(x)$ is the marginal pesticide effectiveness, or simply the density of $G(x)$, whose main properties are $G'(x) > 0$ and $G''(x) < 0$.² Thus, the necessary condition for farmer to maximize profits is given by

$$AG'(x^*) - w = 0 \quad (3.2)$$

The farmer maximizes thus profits by using the optimal quantity of pesticides, x^* , *i.e.* when marginal benefits are equal to marginal cost (pesticide price).

On the other hand, the non-use benefits function is considered to follow a logarithmic path,

$$B(L) = b \ln(L) \quad (3.3)$$

where b is a positive constant. The intuition behind this form is that benefits are an increasing function of wildlife population at a decreasing rate, $B_L > 0$ and $B_{LL} \leq 0$. Eiswerth & Kooten (2009) use such functional form to model non-use benefits of wildlife in a contingent valuation study.

On the other hand, the logistic growth function has been the mainstay to analyze biological populations. Such functional considers the wildlife population net growth, ρ , changing at a rate that is function of current population, in which L first increases until a certain level, and then decreases toward zero as the environmental carrying capacity, K ,

² This last inequality does not hold in all cases. In the next section we will constraint our problem to a determined range where the restriction for the second derivative holds.

of the species is approached.³ We use a logistic growth function to model wildlife population growth:

$$\rho(L) = rL \left(1 - \frac{L}{K}\right) \quad (3.4)$$

where r and K are constants that respectively represent the growth rate and the habitat's maximum biological capacity.

On the other hand, we consider the situation in which pesticide harmfully affects wildlife. The main consequence is the reduction of wildlife population growth rate. We assume a linear relation between pesticide concentration and the reduction of natural growth.⁴ That is, the net wildlife mortality function is proportional to both pesticide concentration and the current wildlife population level:

$$R(\Omega, L) = \mu\Omega L \quad (3.5)$$

In a wide range of pesticide stock, the simplest model is a linear function. This form capture the main features of a kill function, that is, more pesticide kills more wildlife, and as the current population is larger, the number of individuals killed also increases: $R_\Omega > 0$, and $R_L > 0$. Moreover, the available evidence suggests that a dose-response function is unlikely to significantly change over a large range. That is, the straight line model does appear to be appropriate in many cases (Suter, 2007). Note that it also assumes that there is a no-concentration (threshold) of a toxicant below which no adverse effect is expected (Leeuwen & Vermeire, 2007).

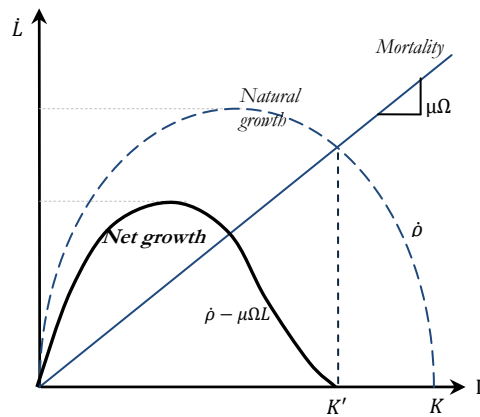


Figure 4.1 Growth-mortality function

In combining both the growth function (3.4) and mortality function (3.5) we obtain the net growth function. In Figure 4.1 the resulting function is represented by the

³ However, a functional form such as the depensatory logistic growth function is used since it explicitly includes a minimum viable population (Conrad, 1999).

⁴ The typically used concentration-response function is a logistic model $R(\Omega) = e^{a+b\Omega}/(1 + e^{a+b\Omega})$ where R is the response of the exposed wildlife population; Ω is the exposure concentration; and a and b constant parameters with no direct biological interpretation.

bold line. Notice that the mortality function reduces both the maximum biological capacity K to K' , as well as the maximum growth rate.

4 Optimal pesticide policy

4.1 Intertemporal optimization

Consider the problem described in (2.4)-(2.8). To solve such maximization problem, we use the maximum principle. The current-value Hamiltonian, where λ_0 and λ_1 denote the co-state variables of wildlife population and pesticide stock respectively, for the control variable x is (suppressing t):

$$\tilde{\mathcal{H}} = [AG(x) - wx] + b \ln L + \lambda_0 \left[rL \left(1 - \frac{L}{K} \right) - \mu\Omega L \right] + \lambda_1 [\alpha x - \beta\Omega] \quad (4.1)$$

Since the control variable, x , is non-negative by assumption (2.7d) along with condition (2.7), the two state variables are always non-negative and the Hamiltonian $\tilde{\mathcal{H}}$ is continuously differentiable with respect to x . To solve this system, *i.e.* to find the optimal trajectories $x^*(t)$, $\Omega^*(t)$, $L^*(t)$, $\lambda_0^*(t)$ and $\lambda_1^*(t)$ we establish the necessary conditions of the control problem:

$$\frac{\partial \tilde{\mathcal{H}}}{\partial x} = AG'(x) - w + \alpha\lambda_1 = 0 \quad (4.2)$$

$$\delta\lambda_0 - \frac{\partial \tilde{\mathcal{H}}}{\partial L} = \dot{\lambda}_0 = \lambda_0 \left[\delta - (r - \mu\Omega) + \frac{2r}{K}L \right] - \frac{b}{L} \quad (4.3)$$

$$\delta\lambda_1 - \frac{\partial \tilde{\mathcal{H}}}{\partial \Omega} = \dot{\lambda}_1 = \lambda_1[\delta + \beta] + \mu\lambda_0 L \quad (4.4)$$

along with

$$\frac{\partial^2 \tilde{\mathcal{H}}}{\partial x^2} = AG''(x) < 0 \quad (4.5)$$

and equation (2.8) are sufficient conditions for an interior solution.

Furthermore, equations (2.4)-(2.7) represent an autonomous problem. Thus the current-value Hamiltonian has the following property as well:

$$\frac{\partial \tilde{\mathcal{H}}^*}{\partial t} = 0 \quad (4.6)$$

The interpretation of the first order conditions is that along the optimal path, according to equation (4.2), the marginal benefits function decrease equals the marginal benefits of decreasing the pesticide deposit across the habitat. The shadow price, $-\lambda_1$, is usually interpreted as the value of diminishing a unit of pesticide stock across the habitat.

The interpretation of the condition (4.2) is that the harmful effects of pesticide on wildlife should take place up to the point where the unit of farmers' benefit is equal to the shadow price of the imputed cost of pollution, which according to condition (4.2) is clearly negative. Furthermore, as the pesticide stock becomes larger the (negative) shadow price should be higher due to increasing damaging costs.

Recall that we further assumed that the pesticide harmfully affected wildlife population beyond the economic optimal population level, so we have thus to make some important remarks. First, if we assume that the initial pesticide concentration, Ω_o , is higher than the optimal, we can expect that $x(t) \leq x_0$. Thus, if the social planner wishes to diminish pesticide concentration through the habitat, she has to follow a path that diminishes the pesticide demand by farmers. It also implies that the net population growth rate, dL/dt , is positive once the policy pesticide is established. That is, the mortality rate is decreasing by diminishing the pesticide stock.

However, in the contrary case, *i.e.*, if the initial pesticide concentration, Ω_o , is smaller than the optimal, it is not possible to state that $x(t) \geq x_0$ for all t . The underlying idea is that the use of pesticides always affects wildlife population, so the regulation of pesticide use might be imposing small taxes.

4.2 Equilibrium analysis

In this section we derive the stationary wildlife population and pesticide concentration across the habitat. In the steady state, $\dot{L} = 0$ and $\dot{\Omega} = 0$, hence from equations (3.4) and (3.5), we obtain the isoclines which respectively are

$$r \left(1 - \frac{\bar{L}}{K}\right) = \mu \bar{\Omega} \quad \text{for} \quad \bar{L} \neq 0 \quad (4.7)$$

$$\bar{\Omega} = \frac{\alpha}{\beta} \bar{x} \quad (4.8)$$

Equation (4.7) simply represents the steady state where growth rate equals mortality rate. Likewise, equation (4.8) can be understood as the equality between pesticide flow and pesticide decay.

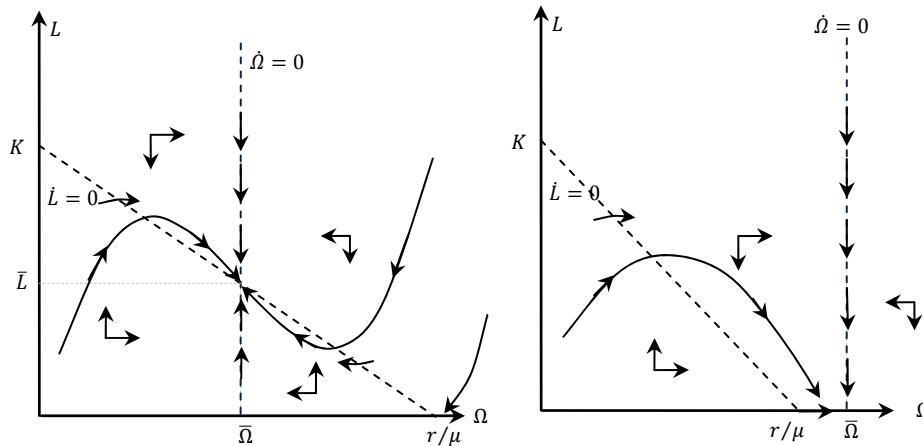


Figure 4.2 Phase-diagram for state-variables dynamics: (a) without extinction, b) extinction

From equation (4.8), the level of the steady state concentration is a vertical line in the $\Omega - L$ space, while equation (4.9) gives as a result a linear decreasing function. We

can see this result in the Figure 4.2. Solving simultaneously the system we obtain the steady state values of wildlife population and pesticide concentration.

$$\bar{L} = K \left(1 - \frac{\mu \alpha}{r \beta} \bar{x} \right) \quad (4.9)$$

Note that another feasible point is also $(0,0)$, but we ignore this trivial equilibrium.

If the equilibrium turnpike $(\bar{\Omega}, \bar{L})$ exists in the positive quadrant, the solution is a global stable node (see Appendix A) toward which the system will tend from any nontrivial point in the positive quadrant. On the contrary, if $\bar{\Omega} > \Omega_{ext}$ the equilibrium will tend to wildlife extinction. Both cases are represented in Figure 4.2, where an arbitrary but typical path is represented by a solid line. Note that at high level of initial wildlife population is even possible to reach the equilibrium at values beyond the extinction level.

In the steady state, $\dot{\lambda}_0 = 0$ and $\dot{\lambda}_1 = 0$. Hence from equations (4.3) and (4.4), we obtain the isoclines which respectively are:

$$[\delta + r - \mu \bar{\Omega}] \bar{\lambda}_0 = \frac{b}{\bar{L}} \quad (4.10)$$

$$[\delta + \beta] \bar{\lambda}_1 = -\mu \bar{L} \bar{\lambda}_0 \quad (4.11)$$

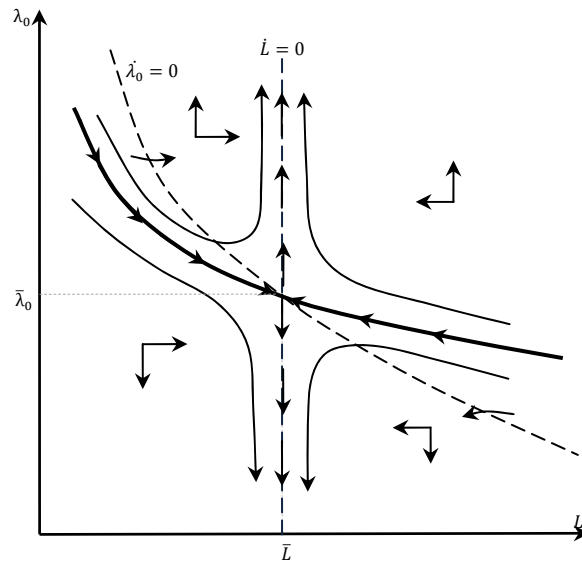


Figure 4.3 Phase-diagram of wildlife population and its shadow price.

Solving the system of simultaneous equations (4.9), (5.3), (4.10) and (4.11) we obtain:

$$\bar{\lambda}_0 = \frac{b}{K \left(1 - \frac{\mu \alpha}{r \beta} \bar{x} \right) \left[\delta + r \left(1 - \frac{\mu \alpha}{r \beta} \bar{x} \right) \right]} \quad (4.12)$$

$$\bar{\lambda}_1 = -\frac{\mu}{(\beta + \delta)} \frac{b}{\left[\delta + r \left(1 - \frac{\mu \alpha}{r \beta} \bar{x} \right) \right]} \quad (4.13)$$

where \bar{x} is the value which solves the resulting equation from equating equations (4.2) and (4.13) for $\dot{\lambda}_1 = 0$:

$$\frac{1}{\alpha} [AG'(\bar{x}) - w] = \frac{\mu}{(\beta + \delta)} \frac{b}{\left[\delta + r \left(1 - \frac{\mu \alpha}{r \beta} \bar{x} \right) \right]} \quad (4.14)$$

However, it is not possible to obtain an explicit solution for \bar{x} . Equations (4.12) and (4.13) have some important properties that will be useful when drawing the phase diagrams:

$$\lim_{\bar{L} \rightarrow 0} \bar{\lambda}_0 = \infty \quad \text{and} \quad \lim_{\bar{L} \rightarrow 0} \bar{\lambda}_1 = -\frac{\mu}{(\beta + \delta)} \frac{b}{\delta} \quad (4.15)$$

We can interpret $-\lambda_1$ as the per unit imputed cost of pollution. When $-\lambda_1$ increases, the imputed cost of pollution increases, which can lead to decrease the pesticide use. Therefore, taking derivative from equation (4.8) we obtain the corresponding slope:

$$\frac{\partial \Omega}{\partial \lambda_1} = \frac{\alpha}{\beta} \frac{\partial x}{\partial \lambda_1} \quad (4.16)$$

From first order condition (4.2), we can obtain x as a function of λ_1 , and the derivative as well:

$$\frac{\partial x}{\partial \lambda_1} = -\frac{\alpha}{AG''(x)} > 0 \quad (4.17)$$

hence, equation (4.16) is non-negative for all $x > 0$. On the other hand, from equations (4.12) and (4.13) we can show that:

$$\frac{\partial \bar{L}}{\partial \bar{\Omega}} < 0 \quad \text{and} \quad \frac{\partial \bar{\lambda}_0}{\partial \bar{\Omega}} > 0 \quad (4.18)$$

Using these equations we can draw the phase-diagram showed in Figure 4.4. As we can see in Figure 4.4, for a given Ω^0 we have to choice λ^0 in regions II and IV such that the pair (Ω^0, λ^0) belongs to the path converging to the turnpike $(\bar{\Omega}, \bar{\lambda})$.

For $\bar{\Omega} < \Omega^0$, we choose λ^0 on the saddle point path in region II. Clearly, the initial control is $x^*(0) = x(\lambda_1^0)$.

So far we have not considered the constraints imposed by the biological system, such as the limit of pesticide concentration that irremediably drives wildlife to extinction. That is the main reason why we do not extensively consider phase-diagram to the right of Ω_{ext} . Likewise, the agricultural production imposes some constraints on the economic range where agricultural production is profitable. Hence, the phase diagram is slightly bounded and modified. We will tackle this problem in the next section, since it entails important consequences to define the optimal pesticide path.

5 Characterization of the optimal taxation policy

We have pointed out that λ_1 can be interpreted as the imputed cost of pollution. Thus, the social planner's problem is how to implement a policy that internalizes such cost. Such policy affects directly the farmers' decision on allocating pesticides to production.

Equations (3.2) and (4.2) are decisive for the pesticide policy. The main difference between both first order conditions is the consideration of the cost of pollution on the

social planner's side when determining the level of pesticide use. The regulator takes into account the external cost of using determined level of pesticides when choosing the socially optimal level of pesticides. The choice implies the consideration on the increased profit of farmers, but also the increased cost of stock pollution that harmfully affects wildlife.

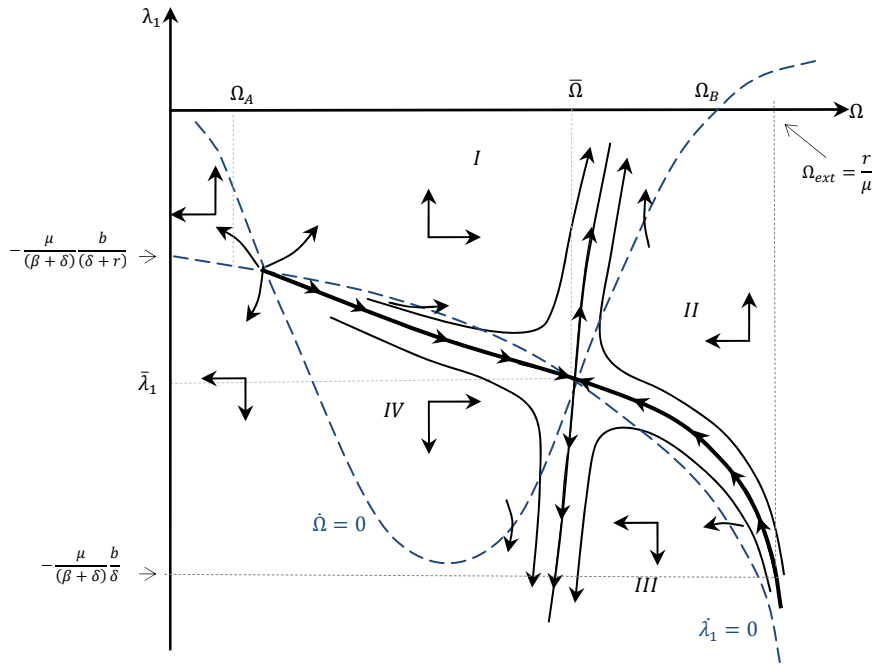


Figure 4.4 Phase-diagram of the concentration-shadow price system.

The economic instrument for the implementation of such a policy can be determined by analyzing the differences. In order to match the decision on the level of pesticides by farmers and the social planner, she has to impose the social cost generated by the choice of using a determined pesticide level.

The economic instrument that can be used is a Pigouvian tax on emissions. With this tax, the farmer maximizes profits when $G'(x) = w + \tau$, hence the optimal dynamic path under this tax is equal to $\tau(t)$ at the time t . Moreover, $\tau = -\alpha\lambda_1$ could be considered as the Pigouvian tax that equalizes social benefits with social costs.

In equation (2.2), w represents the unregulated pesticide price before imposing the tax. Likewise, after imposing a tax, the farmer is assumed to reallocate inputs to obtain the highest profits at the new prices, with $x^*(w + \tau)$, where $\tau \geq 0$ and $x'(w) < 0$. However, we can simplify such problem by considering x as the control variable, so τ will enter the problem just as a dependent variable of pesticide demand, x . Hence, if we know the optimal path of pesticide demand, the tax is directly determined from such function.

5.1 Boundaries for the optimal taxation policy

As mentioned in the foregoing section, we consider taxes as the economic instrument to implement the policy. We only consider positive taxes, that is, we do not consider subsidies. This constraint has important repercussions for the taxation policy. The immediate consequence is on the boundaries of the control variable.

a) Agricultural boundaries

Consider a single farmer; the relevant economic range is mainly delimited by variables relating to the agricultural system. In Figure 5.1 are depicted the well-known relations between revenues, costs, and marginal benefits (recall the properties of $G(x)$ described in section 3.1). The economic range in which a farmer can maximize profits is delimited by $[x_{ban}, x_B]$ for a positive tax. x_B can be defined as the maximum pesticide level that generates the maximum profit before imposing a tax. At this point benefit is $Y(x_B) = Y^{max}$, as we pointed out in (2.2). The imposition of a tax will increase the slope of $C(x)$ moving the cost line upwards. The pesticide level will diminish cost and revenue curve intersect in a single point (Figure 5.1a), where it also turns out that $C'(x) = F'(x)$. The lowest amount of pesticide is thus the level at which the profits are zero, that is, when $Y(x_{ban}) = 0$. As we have assumed linear cost, both bounds can be found when the marginal revenue equalizes marginal cost:

$$\text{Upper bound} \quad F'(x_B) = w, \quad Y(x_B) = 0, \quad Y(x_B) = Y^{max} \quad (5.1)$$

$$\text{Lower bound} \quad \begin{aligned} F'(x_{ban}) &= w + \tau_{ban}, & Y'(x_{ban}) &= 0 \\ F(x_{ban}) &= (w + \tau_{ban})x_{ban}, & Y(x_{ban}) &= 0 \end{aligned} \quad (5.2)$$

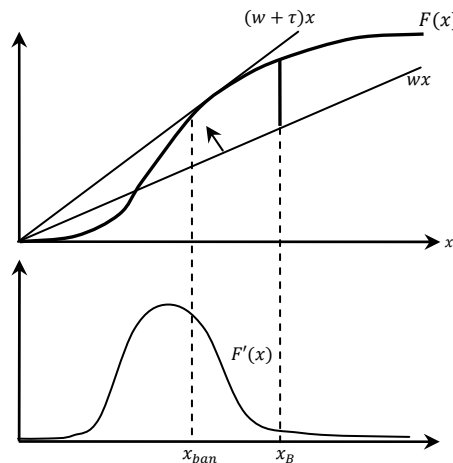


Figure 5.1 a) Revenues and costs, and b) marginal revenues as a function of pesticide use.

If we consider the whole sector, it is not straightforward that we can know such limits. Instead, we assume that the lower limit is positive. From these definitions, *i.e.*, from the point of view of farmers, we can establish the following definition:

Definition 5.1 *In the equilibrium x is at most x_B , since the pesticide use beyond that level decreases benefits as well as increases pesticide pollution. On the other hand, the pesticide level is at least x_{ban} in equilibrium, since it represents the minimum pesticide use level in which profit are non-negative.*

Furthermore, the main properties of effectiveness have been already described in foregoing sections: $G'(x) > 0$ and $G''(x) < 0$.

Hence, the economic range is delimited by these boundaries. In Figure 5.2 these boundaries are depicted as a box whose sides are the horizontal axis, the vertical lines x_{ban} and x_B , as well as horizontal line τ_{ban} . Furthermore, as long as we do not consider dynamic issues, we can even equalize the pesticide level and the pesticide stock according to (5.3), so that these bounds can be equally considered as $\Omega_A = \frac{\alpha}{\beta} x_{ban}$ and $\Omega_B = \frac{\alpha}{\beta} x_B$. It is then obvious that the initial pesticide stock is Ω_0 cannot be larger than Ω_B , since farmers could have used more pesticides than the optimal. However, if there were distorting incentives, such as subsidies, the initial pesticide stock is likely to be higher than Ω_B .

b) Biological boundaries

So far we have described the boundaries determined mainly by the agricultural system. However, the system equilibrium is determined not only by the agricultural parameters but by the biological parameters as well. Such boundaries mainly refer to the extinction level, Ω_{ext} , beyond which wildlife is driven to extinction (recall condition 2.7b). This level is determined by the state equation for wildlife, and whose value is determined by (5.3). Hence, the system's equilibrium has non-negative values $\bar{\Omega}_i \geq 0$ and $\bar{L} \geq 0$ provided that

$$\bar{x} \leq \frac{r\beta}{\mu\alpha} \quad \text{or equally} \quad \Omega_{ext} = \frac{r}{\mu} \quad (5.3)$$

Definition 5.2 *We define $x_{ext} = \frac{r\beta}{\mu\alpha}$ or $\Omega_{ext} = \frac{r}{\mu}$ as the extinction level, *i.e.*, the threshold that would bring wildlife to extinction, since values equal or larger than those ones bring non-positive equilibrium values of L .*

The extinction will take place as the mortality rate exceeds the growth rate when wildlife population approximates zero. This condition can be written as $\lim_{L \rightarrow 0} \frac{\rho(L)}{L} = \mu\Omega_{ext}$. From equations (3.4) and (3.5) we obtain $r = \mu\Omega_{ext}$, which turns out to be equivalent to (5.3).

If equilibrium lies on the interior of the specified boundaries, it implies that it is optimal to use some pesticide forever in the long run. The only point in time when its use is not optimal is at the outset when the pesticide pollution stock is higher (smaller) than $\bar{\Omega}$.

5.2 Optimal trajectory

In this section we describe a general optimal pesticide path. Such policy assumes an interior equilibrium whose initial conditions are also interior. That is, we rule out for the moment that the equilibrium and the initial conditions involve non-negative profits or wildlife extinction.

Proposition 5.1 *There exists an equilibrium $(\bar{L}, \bar{\Omega})$, provided that the marginal private benefits at x_{ban} are equal or higher than the cost of pollution when pollution is close to zero.*

Proof. Consider the bounds described in the Definition 5.1. From (5.2) we know that x_{ban} represents the smallest pesticide use...

$$AG'(x_{ext}) - w \leq \alpha \frac{\mu}{(\beta + \delta)} \frac{b}{r} \quad (5.4)$$

and

$$AG'(x_B) - w \geq \alpha \frac{\mu}{(\beta + \delta)} \frac{b}{(r + \delta)} \quad (5.5)$$

The curves $\hat{\lambda}_1 = 0$ and $\hat{\Omega} = 0$ are decreasing and increasing respectively in Ω . This implies that is sufficient that they intersect in the bounds, or that in the limits of the interval one is bigger than the other one, and in the opposite bound this relation inverts. Equations (5.4) and (5.5) precisely represent this statement. Hence, the set of points in which non-negative profits and non-negative wildlife population is bounded by:

$$S = \{(\Omega, \tau) | \Omega \in [0, \Omega_B] \text{ and } \tau \in [0, \tau_{ban}]\} \quad (5.6)$$

i.e., the feasible region of the $\Omega - \lambda$ space for an interior equilibrium. \square

In fact, we can also state that when $\lim_{L \rightarrow 0} L \frac{\partial B(L)}{\partial L} < \infty$ holds, the inequality (5.4) is not necessary for the equilibrium to exist.

Furthermore, we will use the equivalence $\tau = -\alpha\lambda_1$, instead of simply λ_1 in order to intuitively describe the optimal policy path, as well as to show the implications of the initial conditions to the path.⁵

As we are interested in describing the optimal policy, we will redraw Figure 4.4 by using the interval described in (5.6). The new diagram has some distinctive features. In Figure 5.2, the curve for $\hat{\lambda}_1 = 0$ is asymptotic to the value of $\Omega_{asym} = (\delta + r)/\mu$.

⁵ We will scale up the axis λ_1 by $-\alpha$. The resulting system is then directly related to the tax value. However, in order to avoid confusion, we will continue using the labels $\hat{\lambda}_1 = 0$ and $\hat{\Omega} = 0$. But keep in mind that we are indeed referring to the same curve but with negative sign.

However the feasible solution with a positive wildlife population is to the left of Ω_{ext} . It has important consequences in drawing the phase diagram.

If $\Omega_0 > \Omega_{ext}$, from (2.7b), (4.4) and (4.13), the curve $\dot{\lambda}_1 = 0$ is a horizontal line along the horizontal axis, *i.e.* $\lambda_1 = 0$ converging to x_B . This situation arises when previous incentives (subsidies for example) brought the pesticide concentration beyond the economic range, Ω_B . That means that wildlife is likely to be extinct, so that the equilibrium is simply determined by the agricultural system where profits are maximized. This is the underlying reason for the constraint (2.7). It does not necessarily mean that $L = 0$. However, if L_0 is sufficiently high, it is possible that a positive wildlife population can be even maintained. This situation arises in agricultural systems where high levels of pesticides have been used in short periods.

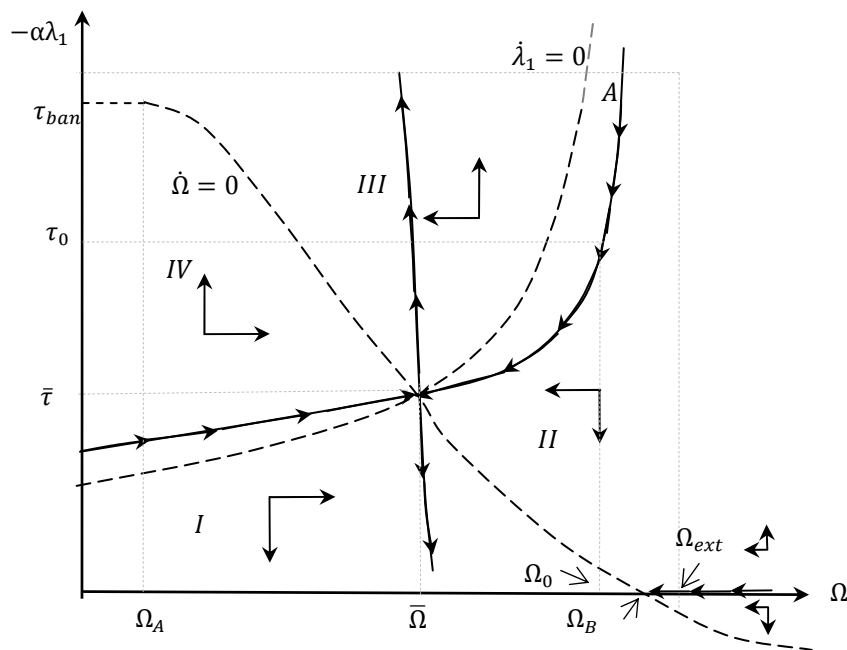


Figure 5.2 Phase-diagram of the control variable and tax.

If the initial Ω_0 state is higher (smaller) than the steady state $\bar{\Omega}$ and within the limits of the interior solution, the optimal tax rate starts at (τ_0, Ω_0) and then decreasing (increasing) at a decreasing rate; it will approach $\bar{\Omega}$ from above (below). Since, $\tau(t)$ is decreasing (increasing), in this case the optimal policy is to tax pesticides at a high (low) rate and gradually decrease (increase) taxing to the turnpike level $(\bar{\Omega}, \bar{\tau})$.

5.3 Key parameters for equilibrium and uniqueness

In this section we describe different cases arising with regard to both the boundaries described in the foregoing section and the system equilibrium. The standard case arises when the equilibrium involves positive profits and positive wildlife population. However, in some cases the equilibrium could inevitably entail either the extinction of wildlife or the complete ban of the pesticide. Both cases are extreme, but have to be considered to get the wide picture of the optimal pesticide policies. For the meantime,

we just describe all cases. We will discuss some conditions (comparative statics) that generate such cases afterwards.

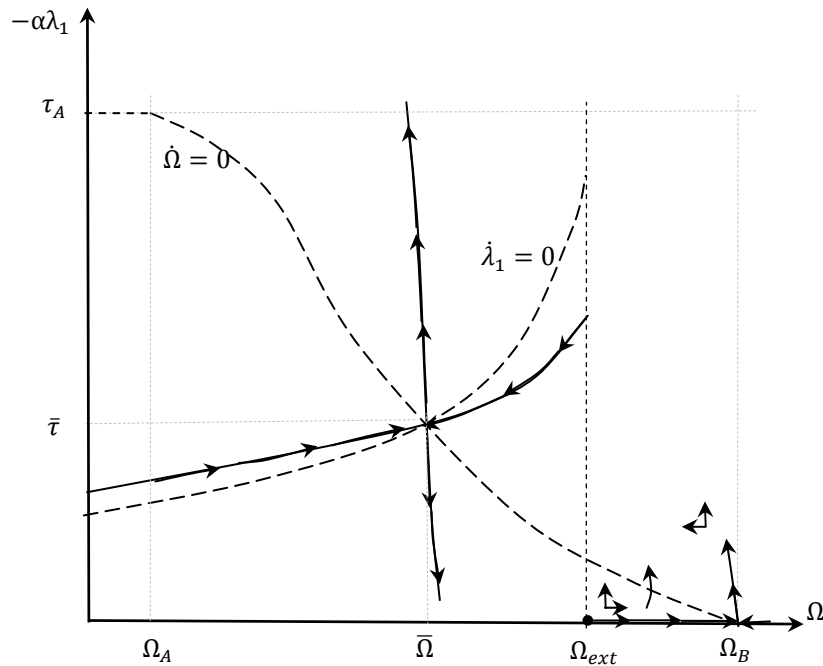


Figure 5.3 Phase-diagram of the control variable and tax.

5.3.1 Interior solutions ($\bar{\Omega} < \Omega_{ext}$)

a) General case with high pesticide pollution stock

We consider the case where $\Omega_B < \Omega_0 < \Omega_{ext}$. As we mentioned in the previous section, this situation arises when, for instance, previous subsidies to pesticides were applied. If this is the case, the first step is to remove these subsidies and then follow the optimal policy as follows.

Figure 5.2 shows that the optimal path (bold line) intersects the horizontal line representing the level τ_A . This point is labeled as A. The significance of this point is crucial. If the tax value along the optimal tax is higher than τ_A , then the optimal control is $\tau^* = \tau_A$, meaning that no pesticide is used, i.e., $x^* = 0$.

If the initial concentration is assumed to lie within the interval (Ω_B, Ω_{ext}) , the optimal pesticide policy is to initially apply the maximum tax, τ_A , to which we will refer as τ_{ban} hereafter, *i.e.* the level at which the use of pesticide is non-profitable any longer (or banned), so that no pesticides are used at this point. We maintain such level until point A, and from this point on we follow the optimal path. This is equally to ban the pesticide, and permit the pesticide stock naturally decay up to a determined level, from which we will follow the optimal path.

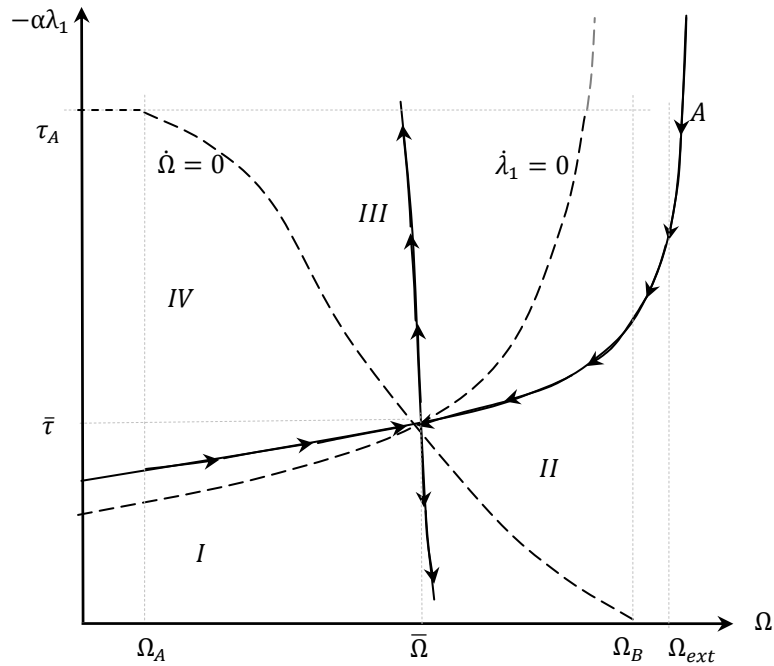


Figure 5.4 Phase-diagram of the control variable and tax.

Figure 5.3 depicts another possibility similar to the previous one. In this situation the optimal path intersects the horizontal line of τ_A in the point marked as A. As Ω_0 intersect to the left of this point, the optimal pesticide is simply to follow the optimal path.

b) Restoring habitat for wildlife

Let now consider the case in which $\Omega_{ext} \leq \Omega_0$. This case is depicted in Figure 5.2 and Figure 5.3. The initial concentration is higher than the extinction level, Ω_{ext} . At first sight, this initial situation would imply no tax, the extinction of wildlife and the maximum profits level at Ω_B as equilibrium. Hence, the optimal policy is to remove subsidies and not to tax pesticides.

However, if the forgone benefits of extinction are sufficiently high, we would be interested in restoring the habitat to provide the conditions to house wildlife, the policy described in section 5.3.1a does make sense. In other words, there is the possibility that some species are still alive, and we are still in time to implement a policy similar to the described in the last paragraphs. That is, given a $L_0 \gg 0$, it is possible to obtain a $L(t) > 0$ in the long term [***provide a proof].

That is, if there is a subsidy, the first step is to remove it, and then to tax at maximum level, τ_{ban} , and then to follow the path presented in a) and then to vary accordingly. It will restore the remaining population or restore the living conditions for wildlife.

c) A pristine habitat

We now draw the attention to the other extreme of the horizontal axis. We consider the case where $\Omega_0 < \Omega_A < \bar{\Omega}$. Figure 5.4 depicts this situation when the initial concentration of the habitat is zero or smaller than the minimum profitable pesticide use. Even when the pesticide stock in the habitat is smaller than Ω_A , the optimal pesticide is to tax it smoothly according to 0. The intuition behind this is that the taxation path slows down the pesticide release rate to the habitat. It diminishes the pollution stock and longer keeps wildlife population at a higher value longer, even when in the long-run they converge to the same equilibrium.

In brief, the initial concentration determines whether the pesticide is initially banned or not. It can be formally stated as follows.

Definition 5.3 *Let $x = \psi(\lambda_1)$ denote the solution of equation (4.2). On account of eq. (5.6), we define the optimal taxation policy as follows*

$$x^*(\tau) = \begin{cases} 0 & : \tau > \tau_{ban} \\ \psi(\lambda_1) & : 0 < \tau \leq \tau_{ban} \\ x_B & : \tau = 0 \end{cases} \quad (5.7)$$

It follows that there exist a unique τ_{ban} such that $\psi(\tau_{ban}) = 0$, $\psi(\tau) \leq 0$ for $\tau \geq \tau_A$ and $\psi(\tau) > 0$ for $0 \leq \tau < \tau_A$.

5.3.2 Special cases. Outer solutions and corner solutions

Case A. Extinction

In this case, the equilibrium level of the pesticide use is less than the minimum pesticide level at which profits are non-negative.

Extinction takes place if the cost of pollution of the first released unit is smaller than the marginal agricultural benefits.

Consider the bounds described in 5.1. From (5.3) we know that x_{ext} represents the threshold...

$$AG'(x_{ext}) - w = AG'\left(\frac{r\beta}{\mu\alpha}\right) - w \geq \frac{\mu}{\delta + \beta} \frac{b}{\delta} \quad (5.8)$$

The proof of this inequality is as in Proposition 5.1.

The interpretation of equation (5.8) is straightforward. If the cost of pollution is not higher than the marginal agricultural benefits, the extinction of a species could be socially optimal.

All optimal paths will inevitably converge to the equilibrium where wildlife is driven to extinction. However, there is a special case when the initial pesticide stock in the habitat is smaller than the Ω_{ext} . The policy is to follow the optimal path until pesticide stock reaches Ω_{ext} . Since the wildlife population is zero from this point on, the

policy is then to remove taxes and let the farmers choose the pesticide level that maximizes profits. [***does it make any sense to follow an optimal path that unavoidably drives wildlife to extinctions? is no-control better? the forgone benefits?]

Case B. Complete pesticide ban

This is an extreme case when the equilibrium is higher than the maximum pesticide tax, τ_{ban} . That is equilibrium lies on any point along this level. In such a case, the optimal pesticide is to set up the fixed tax at τ_{ban} , which implies tha any positive pesticide use is not optimal. Furthermore, in a case where the equilibrium point $(\bar{\Omega}, \bar{\tau})$ lies above τ_{ban} , the pesticide policy is also to ban the pesticide from the beginning. The conditions, for such equilibria will be discussed in the coming section.

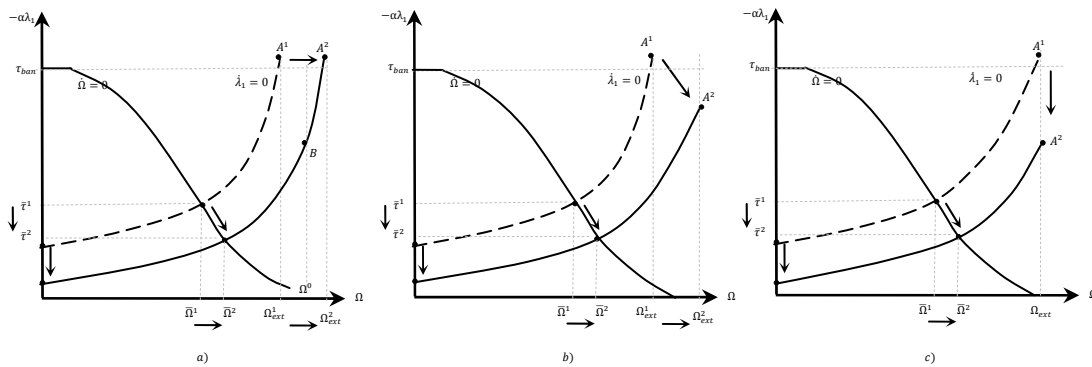


Figure 5.5 Effects of underlying parameters in the optimal path.

5.4 Comparative statics

In this section we discuss the underlying parameters which affect isoclines and the system equilibrium. It is of special importance to examine the changes in the equilibrium solution of optimal control problem when one of the parameters is marginally tightened or relaxed. In particular, the change in the natural decay of pollution (associated to persistency), β , as well as in the parameter associated to mortality, μ . As in many studies (Keeler *et al.*, 1971), we will explore the standard conclusion that higher is the rate of decay, the higher is the level of pollution stock at which the pesticide's is banned.

a) Wildlife-related parameters

As showed in the previous section, the pesticide stock level, at which extinction takes place, plays a crucial role in determining the optimal path. Recall that this level is $\Omega_{ext} = r/\mu$, so it is evident that both parameters directly affect it.

Let take first the effects of an increase (decrease) in the natural growth rate, r . As we can observe in Figure 5.5a, the main effect is the increase (decrease) in the extinction pesticide stock from $\Omega_{ext}^{(1)}$ to $\Omega_{ext}^{(2)}$. A possible explanation for this is that the ratio r/μ represents the net growth (mortality) ratio, which is directly related to the

resilience of wildlife population with regard to external influences as pesticides. That is, an increase in r augments the net growth ratio. Species with high r are more capable to resist high external disruptions. Therefore, an increase (decrease) in r , increases (decreases) the range of resistance to pesticide stock. It also can be interpreted as the decrease of the cost of pollution, since the impact to wildlife is now lesser.

Moreover, it has also as consequence the shift in the curve $\dot{\lambda}_1 = 0$ downwards (upwards). Likewise, the equilibrium point shifts from $(\bar{\Omega}^{(1)}, \bar{\tau}^{(1)})$ to $(\bar{\Omega}^{(2)}, \bar{\tau}^{(2)})$, in such a way that $\bar{\Omega}$ increases (decreases), while $\bar{\tau}$ decreases (increases). Moreover, the intersection of the curve $\dot{\lambda} = 0$ with the vertical line representing Ω_{ext} increases (decreases) from $A^{(1)}$ to $A^{(2)}$.

The main consequence to the optimal pesticide path comes from the shift of the point A . Let consider an initial pesticide stock in the habitat, Ω_0 , that lies within the points $A^{(1)}$ and $A^{(2)}$, as depicted in Figure 5.5a. The initial stock is not affected by r neither by μ , so that if the initial original state is the represented by the dotted line, the optimal policy is likely to impose the maximum tax, τ_{ban} , and then to follow the optimal path. Instead, if the system is described by the bold line, it is likely that the optimal path starts below the point B .

In the case of the mortality-related parameter, μ , it nearly has the same effects on the equilibrium system as r , and hence on the optimal path. The identical explanation for r applies for μ , but in opposite directions. The only difference is in the intersection of the curve $\dot{\lambda}_1 = 0$ and the extinction pesticide stock level, Ω_{ext} . Instead of horizontally moving the point $A^{(1)}$ to $A^{(2)}$, it intersects in a lower point in the vertical direction.

b) Social system-related parameters

In this section we focus on the effects of social system-related underlying parameters in the optimal policy. Unlike the effects of r and μ , the extinction pesticide stock, Ω_{ext} , does not depend upon b and δ . That is, its value remains the unaltered, while the curve $\dot{\lambda}_1 = 0$ moves downwards as b diminishes, or δ increases. In this sense the explanation nearly holds as in the previous case.

The wildlife non-use benefits-related parameter, b , is directly proportional to both the net benefits and the marginal benefits. A decrease (increase) in b results in the shift in from $(\bar{\Omega}^{(1)}, \bar{\tau}^{(1)})$ to $(\bar{\Omega}^{(2)}, \bar{\tau}^{(2)})$, in such a way that $\bar{\Omega}$ increases (decreases), while $\bar{\tau}$ decreases (increases) as shown in Figure 5.5c. That is, a decrease in b means a less value of net benefits from wildlife, and hence the relative value of agricultural output with respect to wildlife benefits is higher. Therefore, pesticide stock equilibrium is higher, since is the net benefits increase as the agricultural output increases and wildlife decreases. Such result can be thus obtained from reducing taxes. The increase in δ has also the same effect; it is possible to interpret this result in the same way in saying that it reduces the wildlife benefits in a distant future.

c) Pesticide stock-related parameters

We discuss now the effects of pesticide stock-related parameters, α and β , both associated to affinity and persistence respectively. The effect of a increase in α , has the same effect as a reduction in β . The main effect is the shift in the curve $\dot{\Omega} = 0$ towards the right-hand side in the $\Omega - \lambda$ space, and the shift in $\dot{\lambda}_1 = 0$ upwards, as depicted in Figure 5.6a. In such a case, the increase of $\bar{\tau}$ is the main effect.

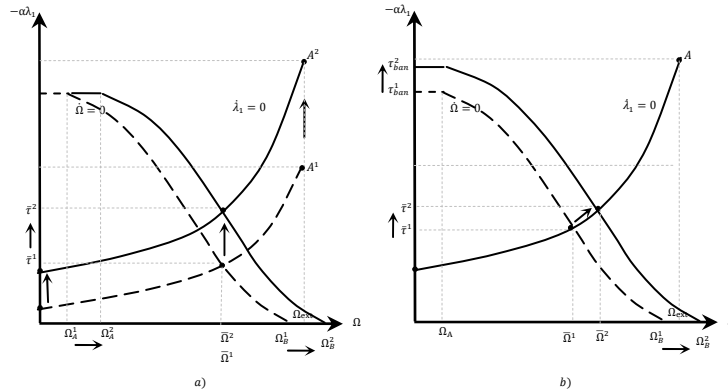


Figure 5.6 Effects of economic parameters in the optimal path.

In any case, the increase in α or decrease in β , that is, the increase in the rate of pesticide flow into the habitat, or the decrease in the pesticide decay, implies a higher pesticide stock equilibrium, $\bar{\Omega}$. It explains the shift in the curve $\dot{\Omega} = 0$ towards the right hand in the $\Omega - \lambda$ space.

Likewise, the curve $\dot{\lambda}_1 = 0$ shifts upwards. Recall that is interpreted as the imputed cost of pollution, λ_1 , that is, as it increases, the tax has to increase as well in order to internalize the costs associated to the externalities generated by pesticides. Moreover, unlike the shift of $\dot{\Omega} = 0$, the movement of $\dot{\lambda}_1 = 0$ upwards diminishes the equilibrium pesticide stock since the imputed cost of pollution is higher.

Therefore, the net effect in $\bar{\Omega}$ is ambiguous. The effect depends upon the marginal values of both curves, given that the net effect can be in either direction.

In Figure 5.6a is depicted the change in $\dot{\lambda}_1 = 0$ as a result of the change in the initial pesticide price. It simply shows that smaller the pesticide price is, higher the tax is to internalize the external cost caused by pesticides.

6 Implementation of the taxation policy

The system previously described could not easily be implemented. However, the optimal path described in the previous section can be considered as a benchmark.

6.1 A fixed control

In the previous section, the implicit assumption was the absence of costs of implementation to establish a system of a dynamic tax that allows controlling pesticide use. However, when we consider the costs of implementation, this system might not be optimal any longer (Toman & Palmer, 1997). For this reason, we study a framework in

which a fixed one-time tax is implemented. Hence, the problem is now to determine the second-best tax level that maximizes the social net benefits studied above. For this purpose we consider the policy implementation in which a fixed tax is established when a stock pollutant causes damages to environment. We draw heavily on Ko *et al.* (1992) in the first section. The main difference with this paper is that we use an explicit relationship between wildlife and pesticides.

As in the previous section, the social planner's problem is to maximize present value of the net social benefits. However, the pesticide use, \hat{x} , is fixed over time:

$$\max_{\hat{x}} \int_0^{\infty} \{Y(\hat{x}) + B(L(t))\} e^{-\delta t} dt \quad (6.1)$$

subject to

$$\frac{\partial L(t)}{\partial t} = rL(t) \left(1 - \frac{L(t)}{K}\right) - \mu\Omega(t)L(t) \quad (6.2)$$

$$\frac{\partial \Omega(t)}{\partial t} = a\hat{x} - \beta\Omega(t) \quad (\alpha, \beta > 0) \quad (6.3)$$

$$L(0) = L_0, L(t) \geq 0 \quad (6.4a)$$

$$\Omega(0) = \Omega_0, \Omega(t) < \Omega_{ext} \quad (6.4b)$$

$$x(0) = x_0, x(t) = \hat{x} \quad (6.4c)$$

$$\delta, \mu, \alpha \text{ and } \beta \in \mathbb{R}^+ \quad (6.4d)$$

The first order condition for an interior solution is:

$$\frac{Y'(\hat{x})}{\delta} = \int_0^{\infty} B'(L) \frac{\partial L}{\partial \Omega} \frac{\partial \Omega}{\partial x} e^{-\delta t} dt \quad (6.5)$$

This equation can be interpreted as the one-time fixed pesticide use, \hat{x} , that balances the flow of marginal benefits to the discounted flow in damages.

In the previous section the pesticide stock asymptotically approaches to the steady state according to the control variable x . In this case, the steady state is given by

$$\hat{\Omega} = \frac{\alpha}{\beta} \hat{x} \quad (6.6)$$

As Ko *et al.* (1992) pointed out, unlike the dynamic framework in section (3.1), the fixed tax, or in our case, the fixed pesticide use depends directly on the initial concentration, Ω_0 . Not surprisingly, both steady-state equilibria thus coincide when the initial concentration is identical in both cases.

6.2 Discrete taxes

The use of multiple discrete taxes can help approaching the first-best solution. We consider the imposition of a fixed tax $\hat{x}^{(i)}$ in the period i , for a finite number of periods, N , *i.e.*, $i = 0, \dots, N$. The cost of switching the taxation regime is denoted by I .

6.3 Intertemporal optimization

The problem of tax phasing in is

$$\begin{aligned}
\max_{\{\hat{x}^{(i)}\}, \{t_1, t_2, \dots, t_N\}, \{N\}} & \int_0^{t_1} \{Y(\hat{x}^{(0)}) + B(L(t))\} e^{-\delta t} dt - I e^{-\delta t_1} \\
& + \int_{t_1}^{t_2} \{Y(\hat{x}^{(1)}) + B(L(t))\} e^{-\delta t} dt - I e^{-\delta t_2} \\
& + \dots \\
& + \int_{t_N}^{\infty} \{Y(\hat{x}^{(N)}) + B(L(t))\} e^{-\delta t} dt
\end{aligned} \tag{6.7}$$

subject to

$$\frac{\partial L(t)}{\partial t} = rL(t) \left(1 - \frac{L(t)}{K}\right) - \mu\Omega(t)L(t) \tag{6.8}$$

$$\frac{\partial \Omega(t)}{\partial t} = a\hat{x}^{(i)} - \beta\Omega(t) \tag{6.9}$$

$$L(0) = L_0, L(t) \geq 0 \tag{6.10a}$$

$$\Omega(0) = \Omega_0, \Omega(t) < \Omega_{ext} \tag{6.10b}$$

$$\hat{x}^{(0)} = x_0 \tag{6.10c}$$

6.3.1 Optimal pesticide policy

We derive the optimal policy by evaluating the optimal level of the tax between any two periods. The maximization of the problem is as in equation (6.1) evaluated between t_i and t_{i+1} :

$$\max_{\{\hat{x}^{(i)}\}} \int_{t_i}^{t_{i+1}} \{Y(\hat{x}^{(i)}) + B(L(t))\} e^{-\delta t} dt \tag{6.11}$$

for $i = 0, \dots, N$. The first order condition is

$$\frac{Y'(\hat{x}^{*(i)})}{\delta} [e^{t_i} - e^{t_{i+1}}] = \int_{t_i}^{t_{i+1}} B'(L) \frac{\partial L}{\partial \Omega} \frac{\partial \Omega}{\partial x} e^{-\delta t} dt \tag{6.12}$$

The t_N is considered the time of the last tax switch. Thus $t_{N+1} = \infty$, *i.e.*, the final tax level, $\hat{x}^{*(N)}$, which lasts from this point on.

6.3.2 Optimal timing of taxation

The problem is to determine the points in time to switch the tax level. The optimization problem is then

$$\begin{aligned}
& \max_{\{t_1, t_2, \dots, t_N\}, \{N\}} \int_0^{t_1} \{Y(\hat{x}^{(0)}) + B(L(t))\} e^{-\delta t} dt - I e^{-\delta t_1} \\
& + \int_{t_1}^{t_2} \{Y(\hat{x}^{*(1)}) + B(L(t))\} e^{-\delta t} dt - I e^{-\delta t_2} \\
& + \dots \\
& + \int_{t_N}^{\infty} \{Y(\hat{x}^{*(N)}) + B(L(t))\} e^{-\delta t} dt \quad i = 1, \dots, N
\end{aligned} \tag{6.13}$$

subject to (6.8)-(6.10), where $\hat{x}^{*(i)}$ are the optimal tax levels in (6.12). The first order conditions for each stage are

$$[Y'(\hat{x}^{*(i-1)}) + B(L) + \delta I] e^{-\delta t_i^*} = 0 \quad i = 1, \dots, N \tag{6.14}$$

For the discrete time t_i^* , we can explicitly solve equation (6.9) *i.e.*, the pesticide concentration at this point in time

$$\Omega(t_i) = \frac{\alpha}{\beta} \sum_{j=1}^i \left[\hat{x}^{*(j-1)} \left(e^{\beta t_j^*} - 1 \right) e^{-\beta \sum_{k=j}^i (t_k^*)} \right] + \Omega_0 e^{-\beta \sum_{j=1}^i (t_j^*)} \quad i = 1, \dots, N \tag{6.15}$$

We draw the attention to the fact that the solution depends explicitly on Ω_0 . However, equation (6.8) cannot be explicitly solved.

Solving equations (6.12) and (6.14) simultaneously for each period, we obtain the optimal values of tax level and switch time, $\hat{x}^{*(i)}$ and t_i^* respectively.

7 Conclusions

The incorporation of an explicit damage function relating pesticide concentration and wildlife population has provided with important insights of the problem. The model allows the incorporation of the change in private agricultural benefits following from controlling pesticide use and the social benefits of using less pesticide in agricultural production.

The regulator then takes into account the external cost of using determined level of pesticides when choosing the socially optimal level of pesticides. The choice implies the consideration on the increased profit of farmers, but also the increased cost of stock pollution that harmfully affects wildlife. In this framework, it is straightforward that taxes turn out to be the economic instrument to implement the policy. Taxes allow matching the decision on the level of pesticides by farmers and the social planner.

We have also fin that the problem of choosing the optimal path is restricted by the constraints imposed by the agricultural and biological systems. Thus, the optimal taxation policy is determined by such bounds. The main result is that the initial concentration is determinant in the optimal path. If the initial concentration is high, the pesticide use has to be highly restricted, and gradually increase its use. On the other

hand, if pesticides are released in a pristine habitat, the optimal path requires a slightly restriction on pesticide use.

We also showed the importance of examining the changes in the equilibrium solution of optimal control problem when one of the parameters is marginally tightened or relaxed. In particular, the change in the natural decay of pollution (associated to persistency), β , as well as in the parameter associated to mortality, μ , where of special importance to further research.

The problem of implementation turned out to be also crucial. The implicit assumption was the absence of costs of implementation to establish a system of a dynamic tax that allows controlling pesticide use. However, when we consider the costs of implementation, this system might not be optimal any longer. The relaxation of such assumption provided with important insights about how to implement a second-best policy.

Further research on farmers' responses to the implementation of discrete taxes should be carried out. The possibility that farmers -expecting future increases in pesticide taxes- could hold inventories, could lead to a situation that they use a higher pesticide amount than the optimally social amount. In such a case the pesticide policy should anticipate such behavior and modify the pesticide policy accordingly.

8 Annexes

Annex A. Proof of existence of a stable node in steady state (general case)

The equation (4.2) represents the steady-state equilibrium of the state equation for the wildlife population. If we implicitly take the derivative of this equation with respect to $\bar{\Omega}$, we obtain the following expression:

$$\frac{\partial L}{\partial \Omega} = \frac{L \frac{\partial R}{\partial \Omega}}{\frac{\partial \rho}{\partial L} - \frac{\rho}{L}} \quad (\text{A.1})$$

The numerator is clearly positive, and the denominator negative. Therefore, equation (4.2) is a downward sloping curve. To check it recall that we assumed that $\rho(L)$ is strictly concave in L , $\rho'(L)$ is monotonically decreasing and $\rho''(L) < 0$.

Since $\rho(L)$ is strictly concave, there is a region where $\rho(L)$ is an increasing function, and then a decreasing function in the adjacent region. The sign of the expression in the increasing function, i.e. where $\partial \rho / \partial L$ is positive we have to prove that

$\partial \rho / \partial L - \rho / L$ is non-positive, that is that

$$\frac{\partial \rho}{\partial L} \leq \frac{\rho}{L} \quad (\text{A.2})$$

At $L(0)$, ρ/L is indeterminate, but if we evaluate the limit when $L \rightarrow 0$ using the L'Hôpital's rule we obtain:

$$\lim_{L \rightarrow 0} \frac{\rho(L)}{L} = \lim_{L \rightarrow 0} \frac{d\rho(L)}{dL} = \frac{d\rho(0)}{dL} \quad (\text{A.3})$$

Therefore, at $L(0)$, $\partial\rho/\partial L = \rho/L$. If we now rearrange equation (A.2) and take the derivative of the equation (A.2) we obtain

$$L \frac{\partial^2 \rho}{\partial L^2} \leq 0 \quad (\text{A.4})$$

Equation (A.4) holds since we assumed that the second derivative is negative. Therefore equations (A.3) and (A.4) prove that the denominator of equation (A.1) is negative.

Annex B. Proof of a saddle point equilibrium

We evaluate the Jacobian matrix at $P(\bar{L}, \bar{\Omega})$, and we obtain the trace and the determinant of the matrix:

$$J[L, \Omega] = \begin{bmatrix} \frac{\partial \rho}{\partial L} - R & -L \frac{\partial R}{\partial \Omega} \\ 0 & -\beta \end{bmatrix} \quad (\text{B.1})$$

the conditions for a stable node must satisfy following conditions:

$$\text{Tr}J \leq 0 \quad (\text{B.2})$$

$$0 \leq \Delta \leq (\text{Tr}J)^2/4 \quad (\text{B.3})$$

we then obtain the trace by using equations (A.1) and (4.2).

$$\text{Tr}J = \left(\frac{\partial \rho}{\partial L} - \frac{\rho}{L} \right) - \beta \quad (\text{B.4})$$

$$\Delta = -\beta \left(\frac{\partial \rho}{\partial L} - \frac{\rho}{L} \right) \quad (\text{B.5})$$

The term in brackets in equation (B.4) is negative, and β is positive by assumption. Hence equation (B.4) satisfies inequality (B.2).

Now, we substitute equations (B.4) and (B.5) in (B.3) and rearranging we obtain

$$\left[\left(\frac{\partial \rho}{\partial L} - \frac{\rho}{L} \right) + \beta \right]^2 \geq 0 \quad (\text{B.6})$$

which holds for all values. Thus, we can verify that and the eigenvalues associated to the Jacobian matrix are real negative.

Annex C. Stability analysis

We now analyze whether such equilibrium point $(\bar{L}, \bar{\Omega})$ is stable. For this purpose we obtain the Jacobian matrix of the system. If at $(\bar{L}, \bar{\Omega})$ its eigenvalues are real negative, thus the interior equilibrium is a stable node.

1. $\bar{L} - \bar{\Omega}$ phase-diagram

We evaluate the Jacobian matrix at $(\bar{L}, \bar{\Omega})$, and we obtain the trace and the determinant of the matrix:

$$J[\bar{L}, \bar{\Omega}] = \begin{bmatrix} r \left(1 - \frac{\bar{L}}{K}\right) - \mu \bar{\Omega} & -\mu \bar{L} \\ 0 & -\beta \end{bmatrix} \quad (\text{C.1})$$

Substituting equations (4.8) and (4.9) we obtain

$$TrJ = \frac{(\alpha \bar{x} \mu - r \beta)}{\beta} - \beta \quad (\text{C.2})$$

$$\Delta = r \beta - \alpha \bar{x} \mu \quad (\text{C.3})$$

By algebraic manipulation and using conditions (5.3) we can verify that the conditions for a global stable node are satisfied:

$$TrJ \leq 0 \quad (\text{C.4})$$

$$0 \leq \Delta \leq (TrJ)^2/4 \quad (\text{C.5})$$

and the eigenvalues associated to the Jacobian matrix are real negative.

2. $\bar{L} - \bar{\lambda}_0$ phase-diagram

We evaluate the Jacobian matrix at $(\bar{L}, \bar{\lambda}_0)$:

$$J[\bar{L}, \bar{\lambda}_0] = \begin{bmatrix} \mu \bar{\Omega} - r & 0 \\ \frac{2r}{K} \bar{\lambda}_1 + \frac{b}{\bar{L}^2} & \delta + r - \mu \bar{\Omega} \end{bmatrix} \quad (\text{C.6})$$

From this system we obtain

$$TrJ = \delta > 0 \quad (\text{C.7})$$

$$\Delta = -(r - \mu \bar{\Omega})(\delta + r - \mu \bar{\Omega}) < 0 \quad (\text{C.8})$$

By algebraic manipulation and using conditions (5.3) we can verify that the conditions for a saddle-point are satisfied:

$$TrJ^2 - 4D > 0 \quad (\text{C.9})$$

$$\Delta \leq 0 \quad (\text{C.10})$$

since eigenvalues associated to the Jacobian matrix are real, distinct and with different signs.

3. $\bar{\Omega} - \bar{\lambda}_1$ phase-diagram

We evaluate the Jacobian matrix at $(\bar{\Omega}, \bar{\lambda}_1)$:

$$J[\bar{\Omega}, \bar{\lambda}_1] = \begin{bmatrix} -\beta & \alpha \frac{dx}{d\lambda_1} \\ 0 & \beta + \delta \end{bmatrix} \quad (\text{C.11})$$

From this system we obtain

$$\text{Tr}J = \delta > 0 \quad (\text{C.12})$$

$$\Delta = -\beta(\beta + \delta) < 0 \quad (\text{C.13})$$

By algebraic manipulation and using conditions (5.3) we can verify that the conditions for a saddle-point are satisfied:

$$\text{Tr}J^2 - 4D > 0 \quad (\text{C.14})$$

$$\Delta \leq 0 \quad (\text{C.15})$$

since eigenvalues associated to the Jacobian matrix are real, distinct and with different signs.

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